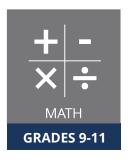# Math Grades 9-11

Participant Packet

Tennessee Department of Education | 2015 Summer Training

MATH

GRADES 9-11

Table of Contents

Module 1

Examining the Beliefs and Knowledge of Effective Mathematics Instruction

Guiding Principles

Across all grade bands and subjects there are common guiding principles for each TNCore training:

- All students are capable of achieving at a high level.

- Students rise to the level of expectation when challenged and supported appropriately.

- Students learn best when they are authentically engaged in their own learning.

- We must continuously improve our effectiveness as teachers and leaders in order to improve student success.

- We must make every minute with our students count with purposeful work and effective instruction.

Norms for Collaboration

- Keep students at the center

- Be present and engaged

- Monitor air time and share your voice

- Challenge with respect

- Stay solutions oriented

- Risk productive struggle

- Balance urgency and patience

Rationale

"All of us who are stakeholders have a role to play and important actions to take if we are finally to recognize our critical need for a world where the mathematics education of our students draws from research, is informed by common sense and good judgment, and is driven by a non-negotiable belief that we must develop mathematical understanding and self confidence in all students."

- NCTM Principles to Action. 2014.

Goals

Participants will:

- Examine productive and unproductive beliefs about mathematics education

- Learn about the different types of knowledge effective mathematics teachers must have

- Connect work from previous TNCore trainings to NCTM's Teaching Practices

Session Activities

Participants will:

- Complete mathematics tasks across the grade band

- Read and discuss NCTM's productive and unproductive beliefs

- Define Mathematical Knowledge for Teaching using a Frayer Model

Overview

Module 1 focuses on the importance of the productive vs. unproductive beliefs (NCTM, 2014) around teaching mathematics that teachers hold. Participants engage in high level tasks and discuss the different mathematical knowledge types (Ball, 2012) and mathematical teaching practices (NCTM, 2014) that support effective and rigorous mathematics instruction.

Practice Problems

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1. A ball is thrown vertically upward at a speed of v ft/sec rises a distance d feet in t seconds, given by $d = 6 + vt - 16t^2$.

    A. Write an equation whose solution is the time it takes a ball thrown at a speed of 88 ft/sec to rise 20 feet.

    B. Write an equation whose solution is the speed with which the ball must be thrown to rise 20 feet in 2 seconds.
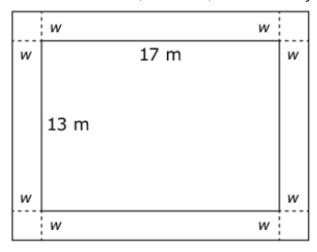
https://www.illustrativemathematics.org/illustrations/437

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2. Which domain would be the most appropriate set to use for a function that predicts the number of household online-devices in terms of the number of people in the household?

    A. Integers

    B. Whole numbers

    C. Irrational numbers

    D. Rational numbers

New York Regents Exam-Common Core Algebra I; http://www.nysedregents.org/
algebraone/ and http://www.nysedregents.org/algebraone/115/algone12015-exam.pdf

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3. A rectangular garden measures 13 meters by 17 meters and has a cement walkway around its perimeter, as shown. The width of the walkway remains constant on all four sides. The garden and walkway have a combined area of 396 square meters.

    A. Write an equation that could be used to help determine the width, w, of the walkway.

    B. Determine the width, in meters, of the walkway.

4. Susie is organizing the printing of tickets for a show her friends are producing. She has collected prices from several printers and these two seem to be the best.

SURE PRINT

Ticket printing

25 tickets for $2

BEST PRINT

Tickets printed

$10 setting up

Plus

$1 for 25 tickets
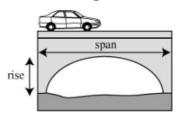
Susie wants to go for the best buy. She doesn't yet know how many people are going to come. Show Susie a couple of ways in which she could make the right decision whatever the number. Illustrate your advice with a couple of examples.

5.

An arch bridge is to be built over a river that is 30 feet wide.

Which span-to-rise ratio would result in an arch in the shape of a semicircle?

Ⓐ 1:2

Ⓑ 2:1

Ⓒ 1:π
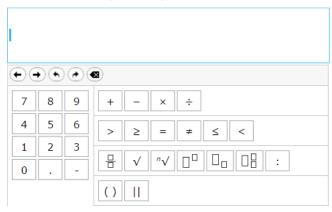
Ⓓ π:1

TNReady/MICA

6.

In early space missions, astronauts ate some foods from tubes shaped like right triangular prisms. The right triangular base of the prisms had a height of 5 centimeters and a base of 1 centimeter. The tubes were 5 centimeters long.

What is the **maximum** number of tubes that could fit in a container in the shape of a rectangular prism with the dimensions 25 cm by 20 cm by 6 cm?

TNReady/MICA

7.
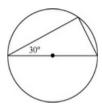
Alexa is given the opportunity to invest $100 in one of two accounts. Account 1 earns 5% annual interest, compounded monthly. The future value, $f(t)$, of Account 2 is determined using the formula $f(t) = 100(1.01)^{4t}$, where t represents the number of years the money is in the account. How do the annual interest rates compare?

Ⓐ The interest rate of Account 1 is 0.25% higher than the rate of Account 2.

Ⓑ The interest rate of Account 2 is 0.60% higher than the rate of Account 1.

Ⓒ The interest rate of Account 1 is 1.00% higher than the rate of Account 2.

Ⓓ The interest rate of Account 2 is 4.00% higher than the rate of Account 1.

TNReady/MICA

8.

A circle and an inscribed triangle are shown. The triangle has the diameter of the circle as one of its sides. The area of the circle shown is 36π square millimeters.

Which statements are true?

Ⓐ The triangle shown is a right triangle.

Ⓑ The triangle shown is a scalene triangle.

Ⓒ The triangle shown has one side length of 3 mm.

Ⓓ The triangle shown has one side length of 18 mm.

Ⓔ The area of the triangle is $18\sqrt{3}$ mm^2 .

TNReady/MICA

NCTM Unproductive and Productive Beliefs

Beliefs About Teaching and Learning Mathematics	
Unproductive Beliefs	**Productive Beliefs**
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Independent/Small Group Time

- What are your biggest takeaways after reading the unproductive and productive beliefs?

- How can the differences in beliefs about teaching and learning mathematics impact students' opportunity to learn?

Mathematical Knowledge for Teaching

What do teachers need to know and be able to do in order to teach effectively? Or, what does effective teaching require in terms of content understanding?
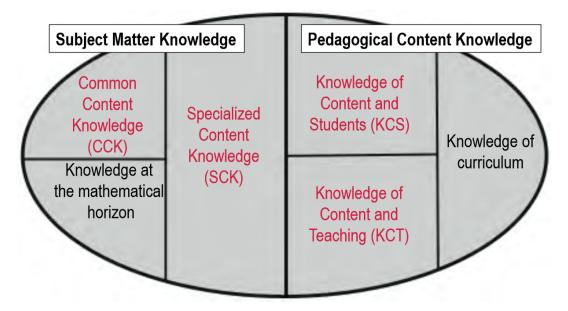
Mathematical knowledge for teaching (MKT) is a kind of professional knowledge of mathematics different from that demanded by other mathematically intensive occupations, such as engineering, physics, accounting, or carpentry. What distinguishes this sort of mathematical knowledge from other knowledge of mathematics is that it is subject matter knowledge needed by teachers for specific tasks of teaching, but still clearly subject matter knowledge. These tasks of teaching depend on mathematical knowledge, and, significantly, they have aspects that do not depend on knowledge of students or of teaching. These tasks require knowing how knowledge is generated and structured in the discipline and how such considerations matter in teaching, such as extending procedures and concepts of whole-number computation to the context of rational numbers in ways that preserve properties and meaning. These tasks also require a host of other mathematical knowledge and skill.

By "mathematical knowledge for teaching," we mean the mathematical knowledge needed to carry out the work of teaching mathematics. It is concerned with the tasks involved in teaching and the mathematical demands of these tasks. By "teaching," we mean everything that teachers must do to support the learning of their students. Clearly we mean the interactive work of teaching lessons in classrooms and all the tasks that arise in the course of that work. But we also mean planning for those lessons, evaluating students' work, writing and grading assessments, explaining the classwork to parents, making and managing homework, attending to concerns for equity, and dealing with the building principal who has strong views about the math curriculum. Each of these tasks, and many others as well, involve knowledge of mathematical ideas, skills of mathematical reasoning, fluency with examples and terms, and thoughtfulness about the nature of mathematical proficiency.

- Kilpatrick, Swafford, & Findell, 2000.

MKT is divided into two domains: subject matter knowledge and pedagogical content knowledge. Each domain includes three categories. Subject matter knowledge includes: common content knowledge, specialized content knowledge, and knowledge of the mathematical horizon. Pedagogical content knowledge includes: knowledge of content and students, knowledge of the curriculum, and knowledge of content and teaching.

Mathematical Knowledge for Teaching

Subject Matter Knowledge

1) **Common Content Knowledge (CCK):** the mathematical knowledge and skill used in settings other than teaching. Teachers need to know the material they teach; they must recognize when their students give wrong answers, or when the textbook gives an inaccurate definition. When teachers write on the board, they need to use terms and notation correctly. In short, they must to be able to do the work that they assign their students. But some of this requires mathematical knowledge and skill that others have as well- thus, it is not special to the work of teaching. By "common," however, we do not mean to suggest that everyone has this knowledge. Rather, we mean to indicate that this is knowledge of a kind used in a wide variety of settings-in other words, not unique to teaching.

2) **Knowledge at the Mathematical Horizon:** Horizon knowledge is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum. First grade teachers, for example, may need to know how the mathematics they teach is related to the mathematics students will learn in third grade to be able to set the mathematical foundation for what will come later. It also includes the vision useful in seeing connections to much later mathematical ideas. Having this sort of knowledge of the mathematical horizon can help in making decisions about how, for example, to talk about the number line.

3) **Specialized Content Knowledge (SCK):** the mathematical knowledge and skill unique to teaching. Close examination reveals that SCK is mathematical knowledge not typically needed for purposes other than teaching. In looking for patterns in student error or in sizing up whether a nonstandard approach would work in general, teachers have to do a kind of mathematical work that others do not. This work involves an uncanny kind of unpacking of mathematics that is not needed or even desirable in settings other than teaching. Many of the everyday tasks of teaching are distinctive to this special work.

Pedagogical Content Knowledge

4) **Knowledge of Content and Students (KCS):** the knowledge that combines knowing about students and knowing about mathematics. Teachers must anticipate what students are likely to think and what they will find confusing. When choosing an example, teachers need to predict what students will find interesting and motivating. When assigning a task, teachers need to anticipate what students are likely to do with it and whether they will find it easy or hard. Teachers must also be able to hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language. Each of these tasks requires an interaction between specific mathematical understanding and familiarity with students and their mathematical thinking. Central to these tasks is knowledge of common student conceptions and misconception about particular mathematical content.

5) **Knowledge of Content and Teaching (KCT):** the knowledge that combines knowing about teaching and knowing about mathematics. Many of the mathematical tasks of teaching require a mathematical knowledge of the design of instruction. Teachers sequence particular content for instruction. They choose which examples to start with and which examples to use to take students deeper into the content. Teachers evaluate the instructional advantages and disadvantages of representations used to teach a specific idea and identify what different methods and procedures afford instructionally. Each of these tasks require an interaction between specific mathematical understanding and an understanding of pedagogical issues that affect student learning.

6) **Knowledge of Curriculum:** is "represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (Shulman, p. 10)."

-Ball, Thames and Phelps: Content Knowledge for Teaching, What Makes It Special? (2008). (n.d.).

-Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing Mathematics for Teaching: Who Knows Mathematics Well Enough To Teach Third Grade, and How Can We Decide?
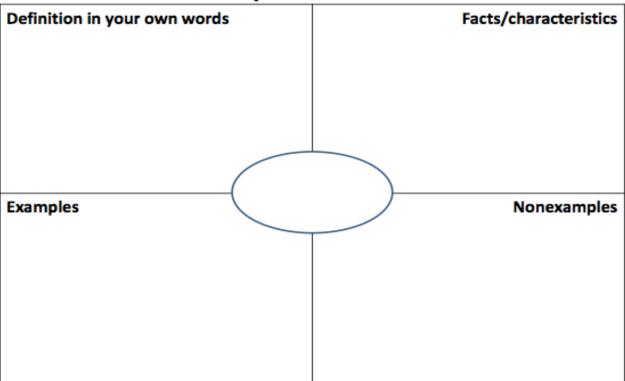
-Shulman, L. S. (1986).Those who understand: Knowledge growth in teaching. *Educational Researcher, I* 5(2), 4-14.

MKT Frayer Model Activity

- Discuss the particular domain of MKT assigned to your group.
- With your group complete Frayer Model for your particular domain on chart paper.
- Be prepared to share your model.

Frayer Model

Definition in your own words	Facts/characteristics
Examples	**Nonexamples**

Small Group Discussion

- What resonates with you about the idea of MKT?

- What did you find new or interesting?

- How do you as a professional mathematics teacher engage in increasing your levels of MKT?

- What will you think about doing differently as a result?

"Teachers who score higher on...measures of mathematical knowledge for teaching produce better gains in student achievement."

- Ball, et al., 2005.

Mathematics Teaching Practices

"Eight Mathematics Teaching Practices provide a framework for strengthening the teaching and learning of mathematics. This research-informed framework of teaching and learning reflects... knowledge of mathematics teaching that has accumulated over the last two decades. The list [below and on the next page] identifies these eight Mathematics Teaching Practices, which represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics"
(Principles to Actions, 2014, p. 9).

Establish mathematics goals to focus learning.	
Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	
Connections to previous trainings	*Tools and resources*

Implement tasks that promote reasoning and problem solving.	
Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.	
Connections to previous trainings	*Tools and resources*

Use and connect mathematical representations.	
Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.	
Connections to previous trainings	*Tools and resources*

Facilitate meaningful mathematical discourse.	
Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.	
Connections to previous trainings	*Tools and resources*

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.	
Connections to previous trainings	*Tools and resources*

Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.	
Connections to previous trainings	*Tools and resources*

Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.	
Connections to previous trainings	*Tools and resources*

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.	
Connections to previous trainings	*Tools and resources*

Small Group Discussion

- What did you find affirming?

- What do you need to refine?

- How do these practices connect to the idea of MKT?

Think about it...

"Effective teaching is the non-negotiable core that ensures that all students learn mathematics at high levels."

                                        -NCTM, Principles to Action Executive Summary.

Reflection

- Why is it necessary to consider productive vs. unproductive beliefs when designing lessons and implementing high-level tasks?

- How does having a deeper awareness of your own content and pedagogical knowledge impact your view of instruction?

- What will you do differently as a result?

- What are you still wondering about?

Module 2
Defining Mathematics Teaching and Learning Through Content Understanding

Rationale

"There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics."

-Lappan & Briars, 1995.

"For many students, making connections between mathematical ideas and the real world is one of the most intriguing and rewarding aspects of the study of mathematics."

- Jacobson, 2014

"Some students leave high school never quite sure of the relevancy of the mathematics they have learned. They fail to see links between school mathematics and the mathematics of everyday life that often requires thoughtful decision making and often complex problem solving."

- Tran & Dougherty, 2014.

Goals

- Synthesize and refine our understanding of teaching and learning
- Consider the mathematical knowledge necessary to teach effectively
- Understand how deep understanding is necessary to support future learning
- Develop a deeper understanding of Modeling Standards
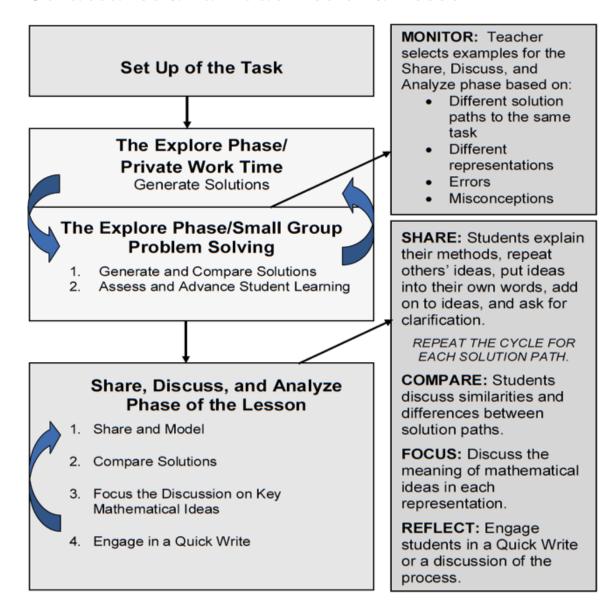
Activities

- Engage in a lesson
- Discuss how the Modeling math practice (MP 4) and the Modeling conceptual category are related
- Discuss teaching strategies and common misconceptions centered around the high school modeling standards

Overview

In Module 2, participants will engage in multiple content-rich tasks designed to further develop their content knowledge and conceptual understanding as teachers of mathematics. Participants will analyze their learning, and reflect on the content through the lens of Mathematical Knowledge for Teaching (Ball, 2012), and the eight Mathematical Teaching Practices (NCTM, 2014).

Structures and Routines of a Lesson

Set Up of the Task	**MONITOR:** Teacher selects examples for the Share, Discuss, and Analyze phase based on: • Different solution paths to the same task • Different representations • Errors • Misconceptions
The Explore Phase/ Private Work Time Generate Solutions **The Explore Phase/Small Group Problem Solving** 1. Generate and Compare Solutions 2. Assess and Advance Student Learning	**SHARE:** Students explain their methods, repeat others' ideas, put ideas into their own words, add on to ideas, and ask for clarification. *REPEAT THE CYCLE FOR EACH SOLUTION PATH.* **COMPARE:** Students discuss similarities and differences between solution paths.
Share, Discuss, and Analyze Phase of the Lesson 1. Share and Model 2. Compare Solutions 3. Focus the Discussion on Key Mathematical Ideas 4. Engage in a Quick Write	**FOCUS:** Discuss the meaning of mathematical ideas in each representation. **REFLECT:** Engage students in a Quick Write or a discussion of the process.

Math Task

Independent Think Time

- Work on the following task privately

Small Group Time

- Compare your work with others in your group
- Consider the different ways to solve the task
- Place your group's thinking on chart paper

Wi-Fi Task

Your firm has been asked to conduct research to determine the economic feasibility to providing free wireless internet (wi-fi) to all Tennesseans throughout the state. It has determined that it would cost on average $100,000 per square mile to provide free, public wi-fi statewide to all Tennesseans.

Your firm is required to estimate the following items in your proposal:

1. A mathematical model for the total cost per person must be provided in the form of an equation, chart, and/or graph with all constants and variables clearly defined. This is required in case one of the variables changes in the future.
2. The total cost of providing free wi-fi to all Tennesseans
3. The cost per person to provide free wi-fi to all Tennesseans.

You are given the following information:

*   The population density of Tennessee is 153.5 persons per square mile.
*   There are 95 counties in Tennessee.
*   The per capita money income during the last 12 months is $24,409.

A map of the state of Tennessee is shown below on a coordinate grid below.

http://www.mapsofworld.com/usa/states/tennessee/tennessee-map.html
http://quickfacts.census.gov/qfd/states/47000.html

Expectations for Whole Group Discussion

- Solution paths will be shared.

- Listen with the goals of:

    - putting the ideas into your own words;

    - adding on to the ideas of others;

    - making connections between solution paths; and

    - asking questions about the ideas shared.

- The goal is to understand the mathematics and to make connections among the various solution paths and representations.

Reflecting on Your Learning

- What about this lesson made it possible for you to learn?

- Which Standards for Mathematical Practice did you have the opportunity to use?

- Where did you see any of the eight teaching practices?

Setting Up The Task
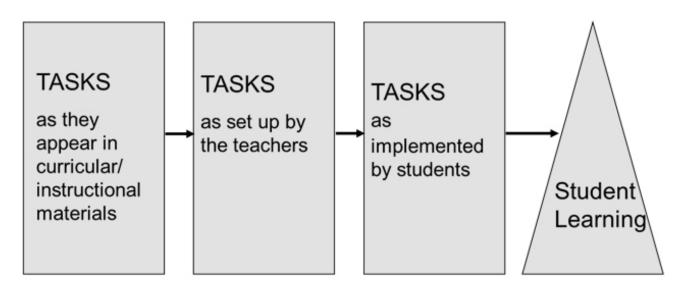
"A task's setup impacts both what students and the teacher are able to achieve during a lesson."

The set-up phase should orient the students to the task in order to ensure all students can engage productively. Here are some key features of the set-up phase:

1. Discuss contextual features.

2. Introduce key mathematical ideas.

3. Establish common language for discussing the mathematics in the task.

4. Maintain the demands of the task.

-Jackson, K., Shahan, E., Gibbons, L., & Cobb, P. (2012). Launching Complex Tasks. *Mathematics Teaching in the Middle School. 18*(1). 24-29.

The Mathematical Tasks Framework

- Stein, M.K., Smith, M.S., Henningsen, M.A., & Silver, E.A. (2000). Implementing standards-based mathematics instruction: A casebook for professional development (p. 4). New York, NY: Teachers College Press.

Factors Associated with Maintenance and Decline of High-Level Cognitive Demands

"Higher-achieving countries implemented a greater percentage of high-level tasks in ways that maintained the demands of the task."

-Stiegler & Hiebert, 2004.

Maintenance

- Scaffolds of student thinking and reasoning provided

- A means by which students can monitor their own progress is provided

- High-level performance is modeled

- A press for justifications, explanations through questioning and feedback

- Tasks build on students' prior knowledge

- Frequent conceptual connections are made

- Sufficient time to explore

Decline

- Problematic aspects of the task become routinized

- Understanding shifts to correctness, completeness

- Insufficient time to wrestle with the demanding aspects of the task

- Classroom management problems

- Inappropriate task for a given group of students

- Accountability for high-level products or processes not expected

Standards of Mathematical Practice: Model with Mathematics

MP4 Model with Mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

High School: Modeling Conceptual Category

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. Some examples of such situations might include:
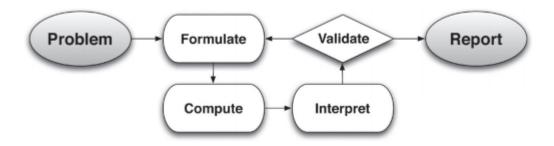
- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them.

Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g. the behavior of polynomials) as well as physical phenomena.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

What is Math Modeling?

Quote	Source
"Because the term modeling is used in a variety of ways in education outside of a mathematical context (e.g., the demonstration stage of a lesson, the construction of some concrete or physical solid, the hopeful emulation of a specific behavior), the term is ripe for misinterpretation."	Imm & Lorber, 2013.
"The end result, a model, is not simply a single solution but a sharable, flexible, and reusable tool."	Imm &Lorber, 2013.
"One practice, modeling with mathematics, focuses on developing the capacity to use math to solve complex problems arising in everyday life. Modeling requires making sense of a problem situation and creating useful mathematical representations for it."	Bostic, 2012.
" *Here's how you can solve the problem'* is quite different from *'this is what I thought about as I solved the problem'* because the former is a polished solution and does not showcase the types of sense making and strategies that a learner may use during the problem-solving process."	Billings, Coffey, Golden, & Wells, 2013.
"Two reasons to model: 1) To gain understanding and 2) To predict or simulate."	Dobson, 2011.
"Quantitative Reasoning is a way to ascribe the mental actions of a student who conceives of a mathematical situation, constructs quantities in that situation, and then relates, manipulates, and uses those quantities to make a problem situation coherent."	Webber, Ellis, Kulow & Ozgur, 2014.
"Models are conceptual structures that represent real or imagined scenarios and relate elements of those situations. Modeling involves building, testing, and applying conceptual representations of phenomena - a practice that is central to learning and doing science and mathematics."	Webber, Ellis, Kulow & Ozgur, 2014 .
"A math model is a math structure that approximates the features of a phenomenon. The process of devising a math model is called math modeling."	NCTM, 1991.
"A math model is a simplification or idealization of the real world using math terms, symbols, and ideas. Math modeling is the application of math to real-world problems."	Albright, 2010.

Simple Tasks

Instructions

- Solve the task.
- Determine if each problem is a modeling task.
- Identify any practice standards the task addresses.
- Identify any content standards (among those listed) that the task addresses.

Task	Problem
A	Debra has six dollars. Robert has 5 dollars. They decide to put their money together and will give n dollars to Stephanie. Write an equation that describes this situation.
B	A coin is a right circular cylinder that has a radius of x inches and a height of y inches. Draw a model that represents this geometric figure.
C	A coin has a radius of x inches and a height of y inches. What is an equation that you can use to find the volume of the cylinder?
D	Write an equation that you can use to solve this problem: To what depth will 50 cubic feet of garden mulch cover a bed of area 20 square feet?
E	What is the amount of money in an account earning 4% interest compounded annually after t years where P is in the initial deposit?
F	Suppose you put $500 in a savings account that is compounded monthly. Your balance B after t years is $B = 500\left(1 + \frac{.05}{12}\right)^{12t}$. (a) Why does the model contain the fraction .05/12? (b) How does this equation reflect the fact that you opened the account with $500? (c) What do the numbers 1 and $\frac{.05}{12}$ represent in the expression $\left(1 + \frac{.05}{12}\right)^{12t}$? (d) What does the "12t" in the equation represent? https://www.illustrativemathematics.org/content-standards/tasks/390
G	The area of a square is a function of the length of its diagonal. What is this function? What is the domain of the function?
H	What is an equation that describes the number of miles travelled in t hours by an automobile at a constant speed of 30 miles per hour?

Will It Overflow? Act I

1. How many meatballs will it take to overflow?

2. What is a number of meatballs you know is too high?

3. What is a number of meatballs you know is too low?

Will It Overflow? Act II

Additional Information:

1. _____

2. _____

3. _____

Homework Task

A recent article claims that there is a relationship between homework and test scores.

- List the key features of the situation.

- Include the assumptions and constraints.

- Simplify the situation.

- Build a math model: (strategy, concepts, data,, variables, constants, etc.).

- Display the model graphically, numerically (table), verbally, and analytically.

Selected High School Modeling Standards

High School: Algebra-Creating Equations*

Create equations that describe numbers or relationships.
A-CED.1
Create equations and inequalities in one variable and use them to solve problems.
Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.2
Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.3
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.

A-CED.4
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R*.

---

* Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (*).

High School: Functions-Interpreting Functions

Interpret functions that arise in applications in terms of the context.

F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*[*]

F-IF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*[*]

F-IF.6

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.[*]

Analyze functions using different representations.

F-IF.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.[*]

> CCSS.Math.Content.HSF.IF.C.7.a
> Graph linear and quadratic functions and show intercepts, maxima, and minima.
>
> CCSS.Math.Content.HSF.IF.C.7.b
> Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
>
> CCSS.Math.Content.HSF.IF.C.7.c
> Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
>
> CCSS.Math.Content.HSF.IF.C.7.d
> (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
>
> CCSS.Math.Content.HSF.IF.C.7.e
> Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

High School: Functions >> Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems.
CCSS.Math.Content.HSF.LE.A.1
Distinguish between situations that can be modeled with linear functions and with exponential functions.

CCSS.Math.Content.HSF.LE.A.1.a
Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

CCSS.Math.Content.HSF.LE.A.1.b
Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

CCSS.Math.Content.HSF.LE.A.1.c
Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

CCSS.Math.Content.HSF.LE.A.2
Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

CCSS.Math.Content.HSF.LE.A.3
Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

CCSS.Math.Content.HSF.LE.A.4
For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.
CCSS.Math.Content.HSF.LE.B.5
Interpret the parameters in a linear or exponential function in terms of a context.

High School: Geometry >> Expressing Geometric Properties with Equations

Use coordinates to prove simple geometric theorems algebraically

CCSS.Math.Content.HSG.GPE.B.7

Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.[*]

High School: Geometry >> Modeling with Geometry

Apply geometric concepts in modeling situations

CCSS.Math.Content.HSG.MG.A.1

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).[*]

CCSS.Math.Content.HSG.MG.A.2

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).[*]

CCSS.Math.Content.HSG.MG.A.3

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).[*]

Modeling in High School Mathematics-Reflection

What Mathematical Modeling Is	What Mathematical Modeling Is Not

Modeling in High School Mathematics – Essential Understandings

- "There are six conceptual categories described in high school mathematics: Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability. The mathematical practice standards are also integrated within each of these conceptual categories. The Modeling category presents a unique structure. No standards are unique to or organized beneath the conceptual category of Modeling; instead, standards are associated with modeling, which are indicated by a star symbol (⋆), are integrated throughout the other categories. Each of these marked standards is meant to indicate a link between mathematics and 'everyday life, work, and decision making' (CCSSI, *p2010c72)."*

  -Schwols & Dempsey, 2012.

- "Mathematical models describe situations in the world, to the surprise of many. *A*lbert Einstein wondered, *"*How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirable appropriate to the objects of reality?*"* This points to the basic reason to model with mathematics and statistics: to understand reality."

  -Progressions for the Common Core State Standards in Mathematics: Modeling, High School, 2013.

- "In the Standards, modeling means using mathematics or statistics to describe (i.e. model) a real world situation and deduce additional information about the situation by mathematical or statistical computation and analysis."

  -Progressions for the Common Core State Standards in Mathematics: Modeling, High School, 2013.

Coda for Mathematics Educators

By Henry Pollak

"Probably 40 years ago, I was an invited guest at a national summer conference whose purpose was to grade the AP Examinations in Calculus. When I arrived, I found myself in the middle of a debate occasioned by the need to evaluate a particular student's solution of a problem. The problem was to find the volume of a particular solid which was inside a unit three-dimensional cube. The student had set up the relevant integrals correctly, but had made a computational error at the end and came up with an answer in the millions. (He multiplied instead of dividing by some power of 10.) The two sides of the debate had very different ideas about how to allocate the ten possible points. Side 1 argued, "He set everything up correctly, he knew what he was doing, he made a silly numerical error, let's take off a point." Side 2 argued, "He must have been sound asleep! How can a solid inside a unit cube have a volume in the millions?!It shows no judgment at all. Let's give him a point."

My recollection is that Side 1 won the argument, by a large margin. But now suppose the problem had been set in a mathematical modeling context. Then it would no longer be an argument just from the traditional mathematics point of view. In a mathematical modeling situation, pure mathematics loses some of its sovereignty. The quality of a result is judged not only by the correctness of the mathematics done within the idealized mathematical situation, but also by the success of the confrontation with reality at the end. If the result doesn't make sense in terms of the original situation in the real world, it's not an acceptable solution.

How would you vote?"

    - Mathematics Modeling Handbook, COMAP, 2012 http://www.comap.com/modelingHB/

Reflection

- In what ways did you think differently about the teaching and learning of mathematics?

- What new insights about the Modeling Standards do you have?

- How do you develop or select instructional strategies to teach modeling?

- How can you relate your learning about modeling to your current work?

Module 3
Maintaining a Focus on Cognitive Rigor in Instruction

Rationale

Students who performed the best on project-based measures of reasoning and problem-solving were in classrooms in which tasks were more likely to be set up <u>and</u> enacted at high levels of cognitive demand.

- Stein & Lane, 1996; Stein, Lane, & Silver,1996.

The success of students was due in part to the high cognitive demand of the curriculum and the teachers' ability to maintain the level of demand during enactment through questioning.

- Boaler & Staples, 2008.

Goals

- Engage NCTM's Teaching Practices with an eye toward maintaining the cognitive rigor of the task
- Consider the importance of teaching as the intersection of content, pedagogy, and student learning

Session Activities

- Review tools that support rigorous task-based instruction
- Work on NCTM's teaching practices with student work
- Gallery walk and refine our ideas

Overview

Module 3 focuses participants back into the importance of intentional instruction through the analysis of a task using tools from previous summer trainings (structures and routines of a lesson, assessing and advancing questions, accountable talk, etc.). Student work will be analyzed, and teachers will react to the work using instructional strategies and effective teaching practices, all while working to maintain the cognitive demand of the task.

Tools to Support Task-Based Instruction

- On the next several pages are tools from previous summers that support selecting and enacting tasks at high levels of cognitive demand.
- In your small group, review your assigned tool and be prepared to give highlights to the whole group.

(Listen as other groups share for new insights if you are familiar or new learnings if you aren't familiar with these tools.)

Tools	New Insights or New Learnings
Task Analysis Guide	
Assessing and Advancing Questions	
Connections Between Representations	
Productive Discussions Through Selecting and Sequencing Student Work	
Accountable Talk Features and Indicators	

Task Analysis Guide

"If we want students to develop the capacity to think, reason, and problem-solve then we need to start with high-level, cognitively complex tasks."

-Stein, M. K. & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. Educational Research and Evaluation, 2 (4), 50-80.

Lower-Level Tasks	Higher-Level Tasks
Memorization Tasks • Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	Procedures With Connections Tasks • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	Doing Mathematics Tasks • Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demands self-monitoring or self-regulation of one's own cognitive processes. • Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Characteristics of Assessing and Advancing Questions

"Asking questions that assess student understanding of mathematical ideas, strategies or representations provides teachers with insights into what students know and can do. The insights gained from these questions prepare teachers to then ask questions that advance student understanding of mathematical concepts, strategies or connections between representations" (NCTM, 2000).
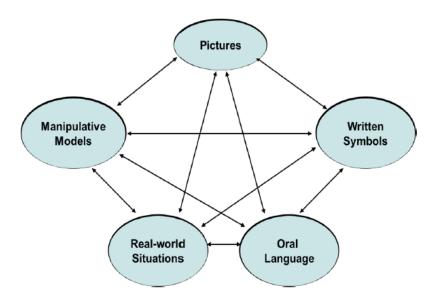
Assessing Questions
- Based closely on the work the student has produced.
- Clarify what the student has done and what the student understands about what s/he has done.
- Provide information to the teacher about what the student understands.

Advancing Questions
- Use what students have produced as a basis for making progress toward the target goal.
- Move students beyond their current thinking by pressing students to extend what they know to a new situation.
- Press students to think about something they are not currently thinking about.

Connections Between Representation

"Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one's self and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling."

Adapted from Lesh, Post, & Behr, 1987

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Orchestrating Productive Discussions

Teachers must "decide what aspects of a task to highlight, how to organize and orchestrate the work of students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge" (NCTM, 2000, p. 19).

Rules of Thumb for Selecting and Sequencing Student Solutions

Based on level of student understanding:

- It is okay to begin by showing incomplete work or work that is not completely clear in order to engage the class in a discussion regarding what else needs to happen to complete or clarify the solution strategy.
- Arrange solutions in order of increasing difficulty with the most complex methods presented last; may be a movement from concrete to abstract.
- Select solutions that illustrate both efficient and inefficient methods so that you can discuss circumstances in which one may be preferable over the other.
- Share at least one completely correct response.
- Don't be afraid to address misconceptions if they are critical to the mathematics being discussed, but stay away from responses that show profound misunderstandings or that do not advance the mathematical discussion.
- Consider individual accomplishments of students (e.g., Is there a student who has not presented in a few days? Is there a student who has done something that is quite unique that would give that student a chance to shine in front of his/her peers?).

Based on the diversity of/similarity of the answers within the classroom:

- Show most frequently used solution methods first to provide entry to all (or the majority) of students.
- Present solutions that show a range of representations (e.g., graphs, tables, equations, diagrams).
- Order solutions (or pair them) so that each solution builds (to the extent possible) on the solution that preceded it.

Based on reaching your mathematical goals:

- Keep the goals and essential understandings in mind and build the discussion so that more students can access these concepts and leave with rich understandings.
- Know that there is more than one way to go about presenting the solutions—have a reason for what you are doing and a goal that your sequencing will target.
- Make sure you get to the generalization (if there is one).

Accountable Talk® Features and Indicators

"Mathematics reform calls for teachers to engage students in discussing, explaining, and justifying their ideas. Although teachers are asked to use students' ideas as the basis for instruction, they must also keep in mind the mathematics that the class is expected to explore" (Sherin, 2000, p. 125).

Accountability to the Learning Community

- Active participation in classroom talk
- Listen attentively
- Elaborate and build on each other's ideas
- Work to clarify or expand a proposition

Accountability to Knowledge

- Specific and accurate knowledge
- Appropriate evidence for claims and arguments
- Commitment to getting it right

Accountability to Rigorous Thinking

- Synthesize several sources of information
- Construct explanations and test understanding of concepts
- Formulate conjectures and hypotheses
- Employ generally accepted standards of reasoning
- Challenge the quality of evidence and reasoning

Think About It...

- Teacher A says, "My students work on problems so they know how to answer these types of problems."

- Teacher B says, "My students work on problems so they can learn the mathematics in these types of problems."

What is the difference between the goals of teacher A and teacher B and why is this significant?
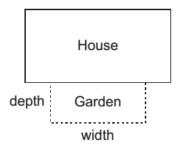
Task-Based Instruction: True or False?

- All tasks must be high-level.

- Accountable talk is only used during a high-level task.

- All high level instructional tasks must have a context.

- Tennessee State Standards require task-based instruction.

- Students never need to engage in low-level tasks.

- Tasks are most effective when they are used to solidify learning.

Effective Uses of a High Level Task	Ineffective Uses of a High Level Task

Vegetable Garden Task

Clariece is fencing in a rectangular garden in her back yard for growing vegetables. She has 18 feet of fence to use. She is using her house to bound one side of the garden, so she only needs to use the fencing for 3 sides, as shown in the diagram below.
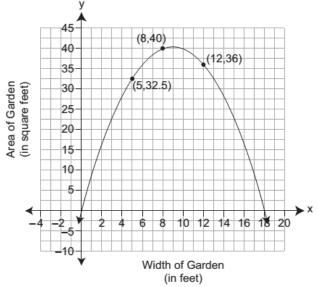
The equation $18 = 2d + w$ can be used to represent the relationship between the depth and width of the garden and the total amount of fencing.

    a. Rearrange the equation so that depth is expressed as a function of width. Show your work.

    b. Solve the equation to find the depth of the vegetable garden if the width is 6.5 feet.

Clariece wants the fence to enclose the largest possible area for her garden. She creates the graph below to represent the relationship between the width of the garden in feet and the area of the garden in square feet.

c. What are the depth and width of the garden with the largest possible area? Explain your reasoning.

Small Group Discussion

• Compare solutions with members in your group.

• What are all the different solution paths your group came up with?
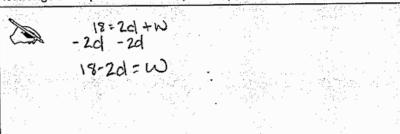
Analyzing Student Work

Independent Think Time

- Carefully examine each student's work.
- What do the students know? Not know? What is the evidence?

Student A
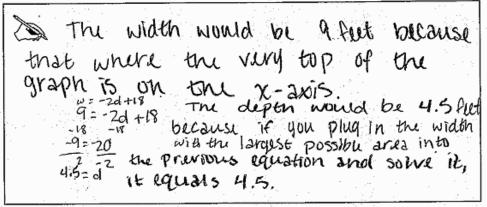
a. Rearrange the equation so that depth is expressed as a function of width. Show your work.

$$18 = 2d + w$$
$$-2d \quad -2d$$
$$18 - 2d = w$$

b. Solve the equation to find the depth of the vegetable garden if the width is 6.5 feet.

$$w = -2d + 18$$
$$6.5 = -2d + 18$$
$$-18 \qquad -18$$
$$\frac{-11.5}{-2} = \frac{-2d}{-2}$$
$$5.75 = d$$
$$5.75 \, ft = d$$

c. What are the depth and width of the garden with the largest possible area? Explain your reasoning.

The width would be 9 feet because that where the very top of the graph is on the x-axis. The depth would be 4.5 feet because if you plug in the width with the largest possible area into the previous equation and solve it, it equals 4.5.

$$w = -2d + 18$$
$$9 = -2d + 18$$
$$-18 \quad -18$$
$$\frac{-9}{2} = \frac{2d}{-2}$$
$$4.5 = d$$
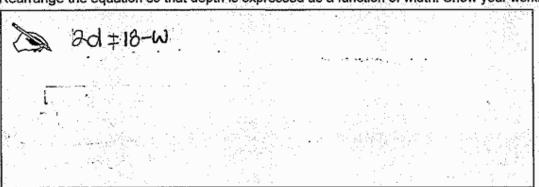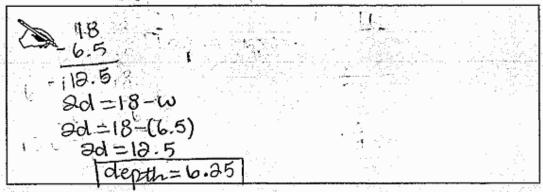
Student B

a. Rearrange the equation so that depth is expressed as a function of width. Show your work.

$2d \neq 18 - w$

b. Solve the equation to find the depth of the vegetable garden if the width is 6.5 feet.

$$\begin{array}{r} 18 \\ -6.5 \\ \hline 12.5 \end{array}$$

$2d = 18 - w$

$2d = 18 - (6.5)$

$2d = 12.5$

$\boxed{depth = 6.25}$

c. What are the depth and width of the garden with the largest possible area? Explain your reasoning.

$\boxed{\begin{array}{l} width = 9 \\ depth = 4.5 \end{array}}$

$2d = 18 - w$

$2d = 18 - 9$

$2d = 9$

$d =$

$$\begin{array}{r} 4.5 \\ 2\overline{)9.0} \\ -8 \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$

Student C

a. Rearrange the equation so that depth is expressed as a function of width. Show your work.

$$18 = 2d + w$$
$$2d + w = 18$$
$$ -w \quad -w$$
$$2d = 18 - w$$

b. Solve the equation to find the depth of the vegetable garden if the width is 6.5 feet.

$$18 = 2d + 6.5$$
$$-6.5 -6.5$$

$$\frac{11.5}{2} = \frac{2d}{2}$$
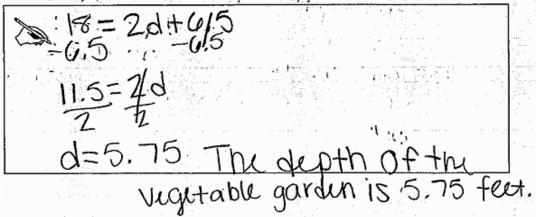
$$d = 5.75 \quad \text{The depth of the vegetable garden is 5.75 feet.}$$

c. What are the depth and width of the garden with the largest possible area? Explain your reasoning.

The hight point of the parabola is when the depth and width of the garden have the largest possible area with a width of 9 feet and a depth of 4.5 feet.
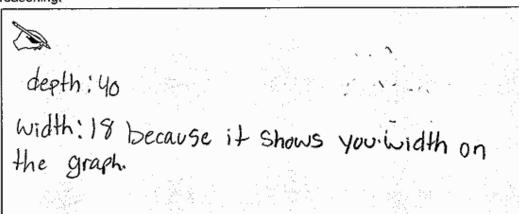
Student D

a. Rearrange the equation so that depth is expressed as a function of width. Show your work.

$$18 = 2d + w$$
$$-w \qquad -w$$
$$w - 18 = 2d$$
$$+18 \qquad +18$$
$$w = 2d + 18$$

b. Solve the equation to find the depth of the vegetable garden if the width is 6.5 feet.

$$w = 2d + 18$$
$$w = 2(6.5) + 18$$
$$w = 31$$

c. What are the depth and width of the garden with the largest possible area? Explain your reasoning.

depth: 40

width: 18 because it shows you width on the graph.
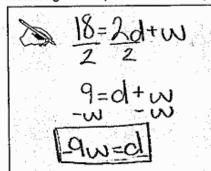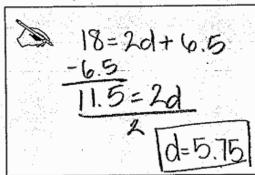
Student E

a.   Rearrange the equation so that depth is expressed as a function of width. Show your work.

$$\frac{18}{2} = \frac{2d+w}{2}$$

$$9 = d + w$$
$$-w \quad\quad -w$$

$$\boxed{-9w = d}$$

b.   Solve the equation to find the depth of the vegetable garden if the width is 6.5 feet.

$$18 = 2d + 6.5$$
$$-6.5$$
$$\overline{11.5 = 2d}$$
$$\quad\quad 2$$
$$\boxed{d = 5.75}$$

c.   What are the depth and width of the garden with the largest possible area? Explain your reasoning.

it would be (8,40) because 40 is on the square side which would make 40² the biggest.

Engaging with the Teaching Practices

With your group, discuss the following questions to complete the chart on the next page.

- What would be your mathematical goal for students as they complete this task?
- For each piece of student work, write one assessing and one advancing question.
- What are the representations that you want to come out in the discussion to support the mathematics?
- Sequence the student work in the order you think it should be shared with the class. What questions might you ask to bring out and connect in support of your goal?
- Are there any other ways to support productive struggle? (i.e. scaffolds, manipulatives)

Structure and Routines of a Lesson

Set Up of the Task	**MONITOR:** Teacher selects examples for the Share, Discuss, and Analyze phase based on: • Different solution paths to the same task • Different representations • Errors • Misconceptions
The Explore Phase/ Private Work Time Generate Solutions **The Explore Phase/Small Group Problem Solving** 1. Generate and Compare Solutions 2. Assess and Advance Student Learning	**SHARE:** Students explain their methods, repeat others' ideas, put ideas into their own words, add on to ideas, and ask for clarification. *REPEAT THE CYCLE FOR EACH SOLUTION PATH.* **COMPARE:** Students discuss similarities and differences between solution paths. **FOCUS:** Discuss the meaning of mathematical ideas in each representation. **REFLECT:** Engage students in a Quick Write or a discussion of the process.
Share, Discuss, and Analyze Phase of the Lesson 1. Share and Model 2. Compare Solutions 3. Focus the Discussion on Key Mathematical Ideas 4. Engage in a Quick Write	

Establish mathematics goals to focus learning.
Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

What are the mathematical goals for this task?

Pose purposeful questions.
Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Write one assessing and advancing question for each student.

Facilitate meaningful mathematical discourse.
Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Sequence the student work for share-discuss-analyze phase of the lesson.
What questions might you ask to problematize the discussion?

Use and connect mathematical representations.
Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Which representations are critical for understanding the mathematics?
How will you ask students to make connections among these representations?

Support productive struggle in learning mathematics.
Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Are there any other ways to support productive struggle? (i.e. scaffolds, manipulatives)

Establish mathematics goals to focus learning	
What are *teachers* doing?	**What are *students* doing?**
• Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit. • Identifying how the goals fit within a mathematics learning progression. • Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. • Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	• Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?) • Using the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices. • Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going. • Assessing and monitoring their own under-standing and progress toward the mathematics learning goals.

Pose purposeful questions	
What are *teachers* doing?	**What are *students* doing?**
• Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. • Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. • Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. • Allowing sufficient wait time so that more students can formulate and offer responses.	• Expecting to be asked to explain, clarify, and elaborate on their thinking. • Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly. • Reflecting on and justifying their reasoning, not simply providing answers. • Listening to, commenting on, and questioning the contributions of their classmates.

Facilitate meaningful mathematical discourse	
What are *teachers* doing?	**What are *students* doing?**
• Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. • Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. • Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. • Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	• Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse. • Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. • Seeking to understand the approach-es used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. • Identifying how different approaches to solving a task are the same and how they are different.

Use and connect mathematical representations	
What are _teachers_ doing?	**What are _students_ doing?**
• Selecting tasks that allow students to decide which representations to use in making sense of the problems. • Allocating substantial instructional time for students to use, discuss, and make connections among representations. • Introducing forms of representations that can be useful to students. • Asking students to make math drawings or use other visual supports to explain and justify their reasoning. • Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. • Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	• Using multiple forms of representations to make sense of and understand mathematics. • Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations. • Making choices about which forms of representations to use as tools for solving problems. • Sketching diagrams to make sense of problem situations. • Contextualizing mathematical ideas by connecting them to real-world situations. • Considering the advantages or suitability of using various representations when solving problems.

Support productive struggle in learning mathematics	
What are _teachers_ doing?	**What are _students_ doing?**
• Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. • Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. • Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. • Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	• Struggling at times with mathematics tasks but knowing that breakthroughs of-ten emerge from confusion and struggle. • Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. • Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. • Helping one another without telling their classmates what the answer is or how to solve the problem.

Gallery Walk

- As a group, examine the work of the other groups.

- Leave targeted, actionable feedback.

- When you return to your group's work, consider revising your work based on the feedback.

- Be prepared to share your thinking with the whole group.

Reflection

- What are some new understandings you have regarding the tools to support task-based instruction?

- What do you believe to be the most critical considerations when reviewing student work? Why?

- Why is it important to consider the mathematical teaching practices when planning to implement a high-level task in your classroom?

- In what ways has this module increased your knowledge in any of the domains of MKT?

Module 4

Implications on Planning and Instruction

Rationale

"The move toward rigor places students squarely at the center of the classroom, where they will grapple with challenging content individually and collaboratively, and where they will be expected to actively demonstrate their learning."

-Marzano and Toth, 2014.

Goals

- Learn about TNReady design for mathematics and instructional implications

- Consider the importance of coherence for learning mathematics at the right grain size

- Examine how student learning is developed through a unit of study

Session Activities

- View blueprints, item types, and calculator policy of TNReady math assessment

- "Unpack" blueprints to consider instructional implications for 2015-2016

- Discuss how the concepts of coherence and grain size are connected to task arc creation and implementation

- Discuss planning units of study as they relate to coherence and the progression of student understanding

Overview

Participants in Module 4 will review the structure and design of the TNReady assessment. Activities in Module 4 engage participants in the process of focusing on the instructional and planning implications of the new assessment, while maintain a strong emphasis on intentionality and coherence and the impact of this focus on unit and lesson planning.

TNReady Note Table

	Noticings	Wonderings	Impact on Planning and Instruction
Math Priorities			
Calculator Policy			
Item Types			
Fluency			
Blueprints			

TNReady Overview

Beginning with the 2015-16 school year, TNReady will provide students, teachers, and parents with more detailed, accurate, and authentic information about each student's progress and achievement in the classroom.

TNReady is more than just a new "TCAP." It is a new way to assess what our students know and what we can do to help them succeed in the future.

TNReady Math Priorities

- Grades 3-8: Focus on fewer concepts – assess those topics in a range of ways
- High School: Strengthen coherence – assess topics in connected ways
- Include authentic assessment of real-life situations
- Support alignment with ACT
- Include calculator-permitted and calculator-prohibited sections at every grade level

Calculator Policy

Two central beliefs:
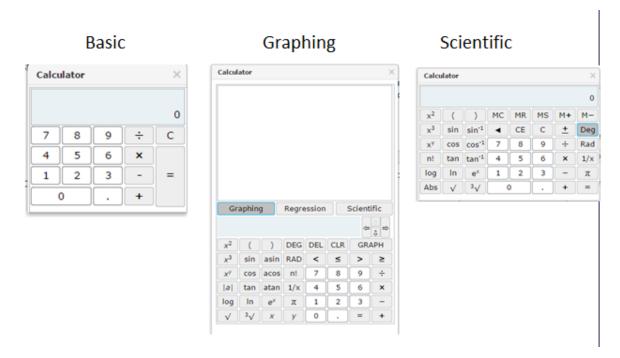
- Calculators are important tools for college and career readiness.
- Students must be able to demonstrate many skills without reliance on calculators.

At all grade levels and in all courses, **TNReady will include both calculator permitted and calculator prohibited sections.**

Examples of permitted and non-permitted calculators, consistent with ACT and other benchmark assessments.

Handhelds are permitted with online testing.

Calculator Types

Basic | Graphing | Scientific

Calculator Policy – Think About it…

- How do the changes in calculator policy impact classroom instruction?

- What considerations will have to be made to ensure students are prepared for this transition?

Math Item Types Overview

There are 6 types of items in Mathematics:

1. Equation

2. Graphic

3. Multiple Choice

4. Multiple Select

5. Performance Tasks (for grade 3-8 only)

6. Technology Enhanced Items

Fluency

Grade	Standard	Expected Fluency
3	**3.**OA.C.7 **3.**NBT.A.2	Multiply/Divide within 100 (Know single digit products from memory) Add/Subtract within 1000
4	4.NBT.B.4	Add/Subtract within 1,000,000
5	5.NBT.B.5	Multi-digit multiplication
6	6.NS.B.2 6.NS.B.3	Multi-digit division Multi-digit decimal operations

Grades 3-6 Fluency standards will only be assessed on Part II of the TNReady assessment. Calculators will not be allowed on fluency items.

Math Blueprints

Blueprint Summary Includes:

- Range of number of items for each part

- Range of percentage for each part

- Total range of number of items

- Total range of percentage for each part

- Percentage of test derived from each cluster

Each blueprint also includes a table that shows:

- What standards are assessed on Part I

- What standards are assessed on Part II

- An overall table for both Part I and Part II

Each of these tables also includes a range of number of items and a range of score points

Grades 3-8 Math Blueprints

- 100% of the content on Part I of the math section will be drawn from the major work of the grade

- 40-60% of the content on Part II of the math section will be drawn from the major work of the grade

- Across both Part I and Part II, 65-75% of the content of the math section will be drawn from the major work of the grade

Content drawn from the major work of the grade is bolded.

High School Math Blueprints

- Clusters have been grouped by category

- Part I includes items that are:

    - Best assessed through equation, graphic and performance tasks

    - Topics that are widely recognized prerequisites for college readiness

    - Topics that need to be treated in a coherent way

- Part II includes all standards with continued focus on questions that draw on the coherence of the standards

- Standards information is available by part

Blueprint Activity

Small Group

- Review the blueprint for your grade level or course.

- Look for the following ideas:

  - Which standards are only assessed in either Part I or Part II?

  - Which standards are assessed on both Part I and Part II?

  - Which sections are assessed most heavily? Least?

Whole Group Discussion

- How will this inform or modify instructional decisions for 2015-2016?

- What activities should we work to engage in to best prepare our students for success?

Grain Size

"Each discipline has a granularity at which its truth is clearest, most coherent. To depart from this grain size in either direction is to depart from the truth."

-Aristole, Ethics.

"[When considering mathematics] proper grain size is the unit at which it makes most sense to organize mathematics for learning."

-Daro, 2013.

Small Group Discussion

- Discuss your noticings and wonderings from the video with your small group.

Grain Size

- Mathematics is simplest at the right grain size

- "Strands" are too big, vague e.g. "numbers"

- Lessons are too small: too many small pieces scattered over the floor, what if some are missing or broken?

- Units are the right size (8-12 a year)

- Stop managing lessons

- Start managing units

Coherence

Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject in which powerful knowledge results from reasoning with a small number of principles such as place value and properties of operations.

-CCSSM K-8 Publishers Criteria, 2012.

Small Group Discussion

- How are the concepts of coherence and grain size connected?

Whole Group Discussion

- Why is it important to consider coherence across standards and concepts when applying the ideas of "grain size" to our planning?

Think About It…

- How do we provide coherent structures around mathematical concepts to ensure students make connections?

Task Arc

[Materials] are coherent if they are: articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential and hierarchical nature of the disciplinary content from which the subject matter derives.

- Schmidt & Houang, 2012, "Curricular Coherence and the Common Core State Standards for Mathematics," Educational Researcher, http://edr.sagepub.com/content/41/8/294, p. 295.

Small Group Discussion

Review the task arc located in your handouts, specifically the arc preview and standards alignment pages. Discuss the following ideas with your small group:

- How does the structure of the task arc support instruction and student learning?

- How do the task arcs support the idea of grain size discussed in the video?

- How could task arcs be utilized in the planning of a unit of study?

Record your group take-aways on chart paper.

Private Think Time

- How do task arcs support the instruction of a group of standards?

Small Group Discussion

- How does the progression of standards within the task arc support student learning? Why is this progression important?

Unit Planning

It is the nature of mathematics that much new learning is about extending knowledge from prior learning to new situations. For this reason, teachers need to understand the progressions in the standards so they can see where individual students and groups of students are coming from, and where they are heading.

- Daro, McCallum, Zimba, 2012.

Private Think Time

- How do you, or teachers in your building, currently work to plan mathematic units of study?

- Do these practices align with the ideas of coherence, grain size, and learning progressions? How or how not?

Small Group Discussion

- How can you utilize the ideas of coherence and grain size to support your own unit planning?

- What resources and tools do you have to support unit planning?

- How can you utilize these tools and resources to ensure student learning develops throughout the unit?

- How do the teaching practices and MKT domains impact your ability to provide coherence and support student learning in your unit plan?

Record your groups thinking on chart paper.
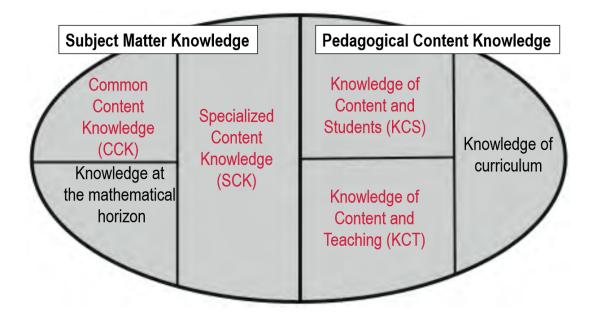
Gallery Walk

- Review other groups ideas on unit planning. Record noticings and wonderings as you walk.

Whole Group Discussion

- How do we work to ensure units of study support student progression through the content in a coherent manner?

Mathematical Knowledge for Teaching

Subject Matter Knowledge		Pedagogical Content Knowledge	
Common Content Knowledge (CCK)	Specialized Content Knowledge (SCK)	Knowledge of Content and Students (KCS)	Knowledge of curriculum
Knowledge at the mathematical horizon		Knowledge of Content and Teaching (KCT)	
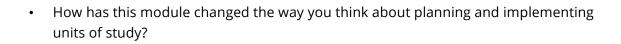
Think About it...

• How does increasing the six domains of subject matter and pedagogical content knowledge areas of MKT support your implementation of the 8 teaching practices as you think about planning, instruction, and assessment?

• Why is it important to consider all six domains when planning a unit of study?

Reflection

- How has this module changed the way you think about planning and implementing units of study?

- What connections can you make between the four modules we have completed?

- What are you still wondering about?

Appendix

Appendix

Tennessee State Standards

Appendix A: Standards for Mathematical Practice

Appendix B: Academic Standards

TNCore Tools

Appendix C: Structure and Routines of a Lesson

Appendix D: Math Accountable Talk® Academic Discussion

Appendix E: Task Analysis Guide

Appendix F: Strategiesf for Modifying Textbook Tasks

TNReady Blueprint

Appendix G: TN Ready Blueprints

Appendix H: Fluency

Appendix I: Item Types

Appendix J: Calculator Policies

Appendix K: Practice Tools

Task Resources

Appendix L: Task Packets

Appendix M: Task Arcs

Progressions

Appendix N: Modeling, High School

Tennessee State Standards

Appendix A
Standards or Mathematical Practice

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Appendix B
Academic Standards

Tennessee's State Mathematics Standards - Core Math I

Domain	Cluster	Standard	Scope and Clarifications
Number and Quantity — Quantities★ (N-Q)	Reason quantitatively and use units to solve problems.	1. Use units as a way to understand problems and to guide the solution of multi-step problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	*There is no additional scope or clarification information for this standard.*
		2. Define appropriate quantities for the purpose of descriptive modeling.	This standard will be assessed in Math I by ensuring that some modeling tasks (involving Math I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described (i.e., a quantity of interest is not selected for the student by the task). For example, in a situation involving data, the student might autonomously decide that a measure of center is a key variable in a situation, and then choose to work with the mean.
		3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	*There is no additional scope or clarification information for this standard.*
Algebra — Seeing Structure in Expressions (A-SSE)	Interpret the structure of expressions.	1. Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	i) Tasks are limited to exponential expressions, including related numerical expressions.
	Write expressions in equivalent forms to solve problems	3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	*There is no additional scope or clarification information for this standard.*
Algebra — Creating Equations★ (A–CED)	Create equations that describe numbers or relationships	1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	i) Tasks are limited to linear or exponential equations with integer exponents. ii) Tasks have a real-world context. iii) In the linear case, tasks have more of the hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).
		2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	i) Tasks are limited to linear equations ii) Tasks have a real-world context. iii) Tasks have the hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).
		3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	*There is no additional scope or clarification information for this standard.*
		4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*	i) Tasks are limited to linear equations ii) Tasks have a real-world context.

Tennessee's State Mathematics Standards - Core Math I

Domain	Cluster	Standard	Scope and Clarifications
Reasoning with Equations and Inequalities (A-REI)	Solve equations and inequalities in one variable	3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	*There is no additional scope or clarification information for this standard.*
	Solve systems of equations	5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	*There is no additional scope or clarification information for this standard.*
		6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	*There is no additional scope or clarification information for this standard.*
	Represent and solve equations and inequalities graphically	10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	*There is no additional scope or clarification information for this standard.*
		11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	i) Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. ii) Finding the solutions approximately is limited to cases where $f(x)$ and $g(x)$ are polynomial.
		12. Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	*There is no additional scope or clarification information for this standard.*

Algebra

Tennessee's State Mathematics Standards – Core Math I

Domain	Cluster	Standard	Scope and Clarifications
Functions — **Interpreting Functions (F-IF)**	Understand the concept of a function and use function notation	1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.	*There is no additional scope or clarification information for this standard.*
		2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	*There is no additional scope or clarification information for this standard.*
		3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = 1, f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*	*There is no additional scope or clarification information for this standard.*
	Interpret functions that arise in applications in terms of the context	4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. The function types listed here are the same as those listed in the Math I column for standards F-IF.6 and F-IF.9.
		5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.
		6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. The function types listed here are the same as those listed in the Math I column for standards F-IF.4 and F-IF.9.
	Analyze functions using different representations	7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	i) Tasks are limited to linear functions.
		9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. The function types listed here are the same as those listed in the Math I column for standards F-IF.4 and F-IF.6.

Tennessee's State Mathematics Standards - Core Math I

Domain	Cluster	Standard	Scope and Clarifications
Building Functions (F-BF)	Build a function that models a relationship between two quantities	1. Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	i) Tasks have a real-world context. ii) Tasks are limited to linear functions and exponential functions with domains in the integers.
		2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★	*There is no additional scope or clarification information for this standard.*
Linear, Quadratic, and Exponential Models★ (F-LE)	Construct and compare linear, quadratic, and exponential models and solve problems	1. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	*There is no additional scope or clarification information for this standard.*
		2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	*There is no additional scope or clarification information for this standard.*
		3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	*There is no additional scope or clarification information for this standard.*
	Interpret expressions for functions in terms of the situation they model	5. Interpret the parameters in a linear or exponential function in terms of a context.	*There is no additional scope or clarification information for this standard.*

Functions

Tennessee's State Mathematics Standards - Core Math I

Domain	Cluster	Standard	Scope and Clarifications
Geometry — Congruence (G-CO)	Experiment with transformations in the plane	1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	*There is no additional scope or clarification information for this standard.*
		2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	*There is no additional scope or clarification information for this standard.*
		3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	*There is no additional scope or clarification information for this standard.*
		4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	*There is no additional scope or clarification information for this standard.*
		5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	*There is no additional scope or clarification information for this standard.*
	Understand congruence in Terms of rigid motions	6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	*There is no additional scope or clarification information for this standard.*
		7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	*There is no additional scope or clarification information for this standard.*
		8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	*There is no additional scope or clarification information for this standard.*
	Prove geometric theorems	9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	*There is no additional scope or clarification information for this standard.*
		10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	*There is no additional scope or clarification information for this standard.*
		11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	*There is no additional scope or clarification information for this standard.*

Tennessee's State Mathematics Standards - Core Math I

Domain	Cluster	Standard	Scope and Clarifications
Statistics and Probability / **Interpreting Categorical and Quantitative Data (S-ID)**	**Summarize, represent, and interpret data on a single count or measurement variable**	1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	*There is no additional scope or clarification information for this standard.*
		2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	*There is no additional scope or clarification information for this standard.*
		3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	*There is no additional scope or clarification information for this standard.*
	Summarize, represent, and interpret Data on two categorical and quantitative variables	5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	*There is no additional scope or clarification information for this standard.*
		6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* c. Fit a linear function for a scatter plot that suggests a linear association.	i) Tasks have real-world context. ii) Tasks are limited to linear functions and exponential functions with domains in the integers.
	Interpret linear models	7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
		8. Compute (using technology) and interpret the correlation coefficient of a linear fit.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*
		9. Distinguish between correlation and causation.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Major Content	Supporting Content	Additional Content

*Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category, and specific modeling standards appear throughout the high school standards indicated with a star (*). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Tennessee's State Mathematics Standards - Core Math II

Number and Quantity

Domain	Cluster	Standard	Scope and Clarifications
The Real Number System (N-RN)	Extend the properties of exponents to rational exponents.	1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	*There is no additional scope or clarification information for this standard.*
		2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	*There is no additional scope or clarification information for this standard.*
	Use properties of rational and irrational numbers.	3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	*There is no additional scope or clarification information for this standard.*
Quantities★ (N-Q)	Reason quantitatively and use units to solve problems.	2. Define appropriate quantities for the purpose of descriptive modeling.	This standard will be assessed in Math II by ensuring that some modeling tasks (involving Math II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., a quantity of interest is not selected for the student by the task). For example, in a situation involving volume of a prism or pyramid, the student might autonomously decide that the area of the base is a key variable in a situation, and then choose to work with that dimension to solve the problem.
The Complex Number System (N-CN)	Perform arithmetic operations with complex numbers.	1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	*There is no additional scope or clarification information for this standard.*
		2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	*There is no additional scope or clarification information for this standard.*
	Use complex numbers in polynomial identities and equations.	7. Solve quadratic equations with real coefficients that have complex solutions.	*There is no additional scope or clarification information for this standard.*

Tennessee's State Mathematics Standards - Core Math II

Algebra

Domain	Cluster	Standard	Scope and Clarifications
Seeing Structure in Expressions (A-SSE)	Interpret the structure of expressions	1. Interpret expressions that represent a quantity in terms of its context.★ b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.*	i) Tasks are limited to quadratic expressions.
Seeing Structure in Expressions (A-SSE)	Write expressions in equivalent forms to solve problems	2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	i) Tasks are limited to quadratic and exponential expressions, including related numerical expressions. ii) Examples: See an opportunity to rewrite $a^2 + 9a + 14$ as $(a+7)(a+2)$. Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53+47)(53-47)$.
Seeing Structure in Expressions (A-SSE)	Write expressions in equivalent forms to solve problems	3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	*There is no additional scope or clarification information for this standard.*
Arithmetic with Polynomials and Rational Expressions (A-APR)	Perform arithmetic operations on polynomials	1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	*There is no additional scope or clarification information for this standard.*
Creating Equations★ (A-CED)	Create equations that describe numbers or relationships	1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	i) Tasks are limited to quadratic and exponential equations. ii) Tasks have a real-world context. iii) In simpler cases (such as exponential equations with integer exponents), tasks have more of the hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).
Creating Equations★ (A-CED)	Create equations that describe numbers or relationships	2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	i) Tasks are limited to quadratic equations ii) Tasks have a real-world context. iii) Tasks have the hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).
Creating Equations★ (A-CED)	Create equations that describe numbers or relationships	4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	i) Tasks are limited to quadratic equations ii) Tasks have a real-world context.
Reasoning with Equations and Inequalities (A-REI)	Understand solving equations as a process of reasoning and explain the	1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	i) Tasks are limited to quadratic equations.
Reasoning with Equations and Inequalities (A-REI)	Solve equations and inequalities in one variable	4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	
Reasoning with Equations and Inequalities (A-REI)	Solve	7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = −3x and the circle $x^2 + y^2 = 3$.*	*There is no additional scope or clarification information for this standard.*

Tennessee's State Mathematics Standards - Core Math II

Functions

Domain	Cluster	Standard	Scope and Clarifications
Interpreting Functions (F-IF)	Interpret functions that arise in applications in terms of the context	4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★	i) Tasks have a real-world context. ii) Tasks are limited to quadratic and exponential functions. The function types listed here are the same as those listed in the Math II column for standards F-IF.6 and F-IF.9.
		5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★	i) Tasks have a real-world context. ii) Tasks are limited to quadratic functions.
		6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	i) Tasks have a real-world context. ii) Tasks are limited to quadratic and exponential functions. The function types listed here are the same as those listed in the Math II column for standards F-IF.4 and F-IF.9.
	Analyze functions using different representations	7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	For F-IF.7a: i) Tasks are limited to quadratic functions. For F-IF.7e i) Tasks are limited to exponential functions.
		8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.*	*There is no additional scope or clarification information for this standard.*
		9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	i) Tasks are limited to on quadratic and exponential functions. ii) Tasks do not have a real-world context. The function types listed here are the same as those listed in the Math II column for standards F-IF.4 and F-IF.6.
Building Functions (F-BF)	Build a function that models a relationship between two quantities	1. Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	For F-BF.1a: i) Tasks have a real-world context. ii) Tasks may involve linear functions, quadratic functions, and exponential functions.
	Build new functions from existing functions	3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	i) Identifying the effect on the graph of replacing $f(x)$ by $f(x) + k$, k $f(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative) is limited to linear and quadratic functions. ii) Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions. iii) Tasks do not involve recognizing even and odd functions. The function types listed in note (ii) are the same as those listed in the Math I and Math II columns for standards F-IF.4, F-IF.6, and F-IF.9.

Tennessee's State Mathematics Standards - Core Math II

Domain	Cluster	Standard	Scope and Clarifications
Similarity, Right Triangles, and Trigonometry (G-SRT)	Understand similarity in terms of similarity transformations	1. Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	*There is no additional scope or clarification information for this standard.*
		2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	*There is no additional scope or clarification information for this standard.*
		3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	*There is no additional scope or clarification information for this standard.*
	Prove theorems involving similarity	4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	*There is no additional scope or clarification information for this standard.*
	Define trigonometric ratios and solve problems involving right triangles	5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	*There is no additional scope or clarification information for this standard.*
		6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	*There is no additional scope or clarification information for this standard.*
		7. Explain and use the relationship between the sine and cosine of complementary angles.	*There is no additional scope or clarification information for this standard.*
		8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★	*There is no additional scope or clarification information for this standard.*
Geometric Measurement and Dimension (G-GMD)	Explain volume formulas and use	1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	*There is no additional scope or clarification information for this standard.*
		3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	*There is no additional scope or clarification information for this standard.*

Geometry

Tennessee's State Mathematics Standards - Core Math II

Domain	Cluster	Standard	Scope and Clarifications
Interpreting Categorical and Quantitative Data (S-ID)	Summarize, represent, and interpret data on two categorical and quantitative variables	6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals.	For S-ID.6a: i) Tasks have real-world context. ii) Tasks are limited to quadratic functions. For S-ID.6b: i) Tasks have a real-world context. ii) Tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.
Conditional Probability and the Rules of Probability (S-CP)	Understand independence and conditional probability and use them to interpret data	1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	*There is no additional scope or clarification information for this standard.*
		2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	*There is no additional scope or clarification information for this standard.*
		3. Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	*There is no additional scope or clarification information for this standard.*
		4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*	*There is no additional scope or clarification information for this standard.*
		5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*	*There is no additional scope or clarification information for this standard.*
	Use the rules of probability to compute probabilities of compound events in a uniform probability model	6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.	*There is no additional scope or clarification information for this standard.*
		7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.	*There is no additional scope or clarification information for this standard.*

Statistics and Probability

Major Content	Supporting Content	Additional Content

*Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category, and specific modeling standards appear throughout the high school standards indicated with a star (*). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Tennessee's State Mathematics Standards - Core Math III

Domain	Cluster	Standard	Scope and Clarifications
Quantities (N-Q) *(Number and Quantity)*	Reason quantitatively and use units to solve problems.	2. Define appropriate quantities for the purpose of descriptive modeling.	This standard will be assessed in Math III by ensuring that some modeling tasks (involving Math III content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., a quantity of interest is not selected for the student by the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude.
Seeing Structure in Expressions (A-SSE) *(Algebra)*	Interpret the structure of expressions	2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	i) Tasks are limited to polynomial and rational expressions. ii) Examples: see $x^2 - y^2$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. In the equation $x^2 + 2x + 1 + y^2 = 9$, see an opportunity to rewrite the first three terms as $(x+1)^2$, thus recognizing the equation of a circle with radius 3 and center $(-1, 0)$. See $(x^2 + 4)/(x^2 + 3)$ as $((x^2+3) + 1)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.
	Write expressions in equivalent forms to solve problems	4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★	*There is no additional scope or clarification information for this standard.*
Arithmetic with Polynomials and Rational Expressions (A-APR)	Understand the relationship between zeros and factors of polynomials	2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	*There is no additional scope or clarification information for this standard.*
		3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	*There is no additional scope or clarification information for this standard.*
	Use polynomial identities to solve Problems	4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*	*There is no additional scope or clarification information for this standard.*
	Rewrite rational expressions	6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	*There is no additional scope or clarification information for this standard.*

Tennessee's State Mathematics Standards - Core Math III

Domain	Cluster	Standard	Scope and Clarifications
Creating Equations★ **(A-CED)**	Create equations that describe numbers or relationships	1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	i) Tasks are limited to simple rational or exponential equations ii) Tasks have a real-world context.
		2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	i) Tasks are limited to simple polynomial, rational, or exponential equations ii) Tasks have a real-world context.
Reasoning with Equations and Inequalities (A-REI)	Understand solving equations as a process of reasoning and explain the reasoning	1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	i) Tasks are limited to simple rational or radical equations.
	Represent and solve equations	2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	*There is no additional scope or clarification information for this standard.*
		11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	i) Tasks may involve any of the function types mentioned in the standard.

Algebra

Tennessee's State Mathematics Standards - Core Math III

Domain	Cluster	Standard	Scope and Clarifications
Interpreting Functions (F-IF)	**Interpret functions that arise in applications in terms of the context**	4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★	i) Tasks have a real-world context. ii) Tasks may involve polynomial, logarithmic, and trigonometric functions. The function types listed here are the same as those listed in the Math III column for standards F-IF.6 and F-IF.9.
		6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★	i) Tasks have a real-world context. ii) Tasks may involve polynomial, logarithmic, and trigonometric functions. The function types listed here are the same as those listed in the Math III column for standards F-IF.4 and F-IF.9.
	Analyze functions using different representations	7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	For F-IF.7e: i) Tasks are limited to logarithmic and trigonometric functions.
		9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	i) Tasks have a real-world context. ii) Tasks may involve polynomial, logarithmic, and trigonometric functions. The function types listed here are the same as those listed in the Math III column for standards F-IF.4 and F-IF.6.

Functions

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Tennessee's State Mathematics Standards - Core Math III

Domain	Cluster	Standard	Scope and Clarifications
Building Functions (F-BF)	**Build new functions from existing functions**	3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	i) Tasks are limited to exponential, polynomial, logarithmic, and trigonometric functions. ii) Tasks may involve recognizing even and odd functions. The function types listed in note (i) are the same as those listed in the Math III column for standards F-IF.4, F-IF.6, and F-IF.9.
Linear, Quadratic, and Exponential Models* (F-LE)	**Construct and compare linear, quadratic, and exponential models and solve problems**	4. Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) = 2 x^3 or f(x) = (x+1)/(x-1) for x ≠ 1.*	*There is no additional scope or clarification information for this standard.*
		4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	*There is no additional scope or clarification information for this standard.*
Trigonometric Functions (F-TF)	**Extend the domain of trigonometric functions using the unit circle**	1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	*There is no additional scope or clarification information for this standard.*
		2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	*There is no additional scope or clarification information for this standard.*
	Model periodic phenomena with trigonometric functions	5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★	*There is no additional scope or clarification information for this standard.*
	Prove and trigonometric identities	8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	*There is no additional scope or clarification information for this standard.*

Functions

Tennessee's State Mathematics Standards - Core Math III

Domain	Cluster	Standard	Scope and Clarifications
Congruence (G-CO)	**Make geometric constructions**	12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment ;and constructing a line parallel to a given line through a point not on the line.	There is no additional scope or clarification information for this standard.
		13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	There is no additional scope or clarification information for this standard.
Circles (G-C)	**Understand and apply theorems about circles**	1. Prove that all circles are similar.	There is no additional scope or clarification information for this standard.
		2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	There is no additional scope or clarification information for this standard.
		3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	There is no additional scope or clarification information for this standard.
	Find arc lengths and areas of sectors of circles	5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	There is no additional scope or clarification information for this standard.

Geometry

Tennessee's State Mathematics Standards - Core Math III

Geometry

Domain	Cluster	Standard	Scope and Clarifications
Expressing Geometric Properties with Equations (G-GPE)	Translate between the geometric description and the equation for a conic section	1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	*There is no additional scope or clarification information for this standard.*
		2. Derive the equation of a parabola given a focus and directrix.	*There is no additional scope or clarification information for this standard.*
	Use coordinates to prove simple geometric theorems algebraically	4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*	*There is no additional scope or clarification information for this standard.*
		5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	*There is no additional scope or clarification information for this standard.*
		6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	*There is no additional scope or clarification information for this standard.*
		7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	*There is no additional scope or clarification information for this standard.*
Geometric Measurement and Dimension (G-GMD)	Visualize relationships between two-dimensional and three-dimensional objects	4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	*There is no additional scope or clarification information for this standard.*
Modeling with Geometry (G-MG)	Apply geometric concepts in modeling situations	1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★	*There is no additional scope or clarification information for this standard.*
		2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★	*There is no additional scope or clarification information for this standard.*
		3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★	*There is no additional scope or clarification information for this standard.*

Tennessee's State Mathematics Standards – Core Math III

Statistics and Probability

Domain	Cluster	Standard	Scope and Clarifications
Interpreting Categorical and Quantitative Data (S-ID)	Summarize, represent, and interpret data on a single count or measurement variable	4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	*There is no additional scope or clarification information for this standard.*
	Summarize, represent, and interpret data on two categorical and quantitative variables	6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals.	For S-ID.6a: i) Tasks have a real-world context. ii) Tasks are limited to exponential functions with domains not in the integers and trigonometric functions.
Making Inferences and Justifying Conclusions (S-IC)	Understand and evaluate random processes underlying statistical experiments	1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	*There is no additional scope or clarification information for this standard.*
		2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model!?*	*There is no additional scope or clarification information for this standard.*
	Make inferences and justify conclusions from sample surveys, experiments, and observational studies	3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	*There is no additional scope or clarification information for this standard.*
		4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	*There is no additional scope or clarification information for this standard.*
		5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	*There is no additional scope or clarification information for this standard.*
		6. Evaluate reports based on data.	*There are no assessment limits for this standard. The entire standard is assessed in this course.*

Major Content	Supporting Content	Additional Content

*Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category, and specific modeling standards appear throughout the high school standards indicated with a star (*). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Tennessee's State Mathematics Standards - Algebra I

Domain	Cluster	Standards	Scope and Clarifications
The Real Number System (N-RN) (Number and Quantity)	Use properties of rational and irrational numbers	3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	There is no additional scope or clarification information for this standard.
Quantities★ (N-Q) (Number and Quantity)	Reason quantitatively and use units to solve problems.	1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	There is no additional scope or clarification information for this standard.
		2. Define appropriate quantities for the purpose of descriptive modeling.	This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described (i.e., a quantity of interest is not selected for the student by the task). For example, in a situation involving data, the student might autonomously decide that a measure of center is a key variable in a situation, and then choose to work with the mean.
		3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	There is no additional scope or clarification information for this standard.
Seeing Structure in Expressions (A-SSE) (Algebra)	Interpret the structure of expressions	1. Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.	There is no additional scope or clarification information for this standard.
		2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.	i) Tasks are limited to numerical expressions and polynomial expressions in one variable. ii) Examples: Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53+47)(53-47)$. See an opportunity to rewrite $a^2 + 9a + 14$ as $(a+7)(a-2)$.
	Write expressions in equivalent forms to solve problems	3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.	i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation. ii) Tasks are limited to exponential expressions with integer exponents.

Adopted by the Tennessee State Board of Education, October 2013

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Tennessee's State Mathematics Standards - Algebra I

Domain	Cluster	Standard	Scope and Clarifications
Arithmetic with Polynomials and Rational Expressions (A–APR)	Perform arithmetic operations on polynomials	1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	*There is no additional scope or clarification information for this standard.*
	Understand the relationship between zeros and factors of Polynomials	3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	i) Tasks are limited to quadratic and cubic polynomials in which linear and quadratic factors are available. For example, find the zeros of $(x - 2)(x^2 - 9)$.
Creating Equations★ (A–CED)	Create equations that describe numbers or relationships	1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	i) Tasks are limited to linear, quadratic, or exponential equations with integer exponents.
		2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	*There is no additional scope or clarification information for this standard.*
		3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	*There is no additional scope or clarification information for this standard.*
		4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*	*There is no additional scope or clarification information for this standard.*

Algebra

Tennessee's State Mathematics Standards – Algebra I

Algebra

Reasoning with Equations and Inequalities (A-REI)

Understand solving equations as a process of reasoning and explain the reasoning

Standard	Scope and Clarifications
1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	i) Tasks are limited to quadratic equations.

Solve equations and inequalities in one variable

Standard	Scope and Clarifications
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	*There is no additional scope or clarification information for this standard.*
4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	For A-REI.4b: i) Tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require the student to recognize cases in which a quadratic equation has no real solutions. *Note, solving a quadratic equation by factoring relies on the connection between zeros and factors of polynomials (cluster A-APR.B). Cluster A-APR.B is formally assessed in A2.*

Solve systems of equations

Standard	Scope and Clarifications
5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	*There is no additional scope or clarification information for this standard.*
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	i) Tasks have a real-world context. ii) Tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).

Represent and solve equations and inequalities graphically

Standard	Scope and Clarifications
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	*There is no additional scope or clarification information for this standard.*
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	i) Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. ii) Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial functions.
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	*There is no additional scope or clarification information for this standard.*

Tennessee's State Mathematics Standards - Algebra I

Functions

Interpreting Functions (F-IF)

Understand the concept of a function and use function notation

Standard	Scope & Clarification
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	*There is no additional scope or clarification information for this standard.*
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	*There is no additional scope or clarification information for this standard.*
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*	i) This standard is part of the Major work in Algebra I and will be assessed accordingly.

Interpret functions that arise in applications in terms of the context

Standard	Scope & Clarification
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. *Compare note (ii) with standard F-IF.7. The function types listed here are the same as those listed in the Algebra I column for standards F-IF.6 and F-IF.9.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★	*There is no additional scope or clarification information for this standard.*
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. *The function types listed here are the same as those listed in the Algebra I column for standards F-IF.4 and F-IF.9.*

Tennessee's State Mathematics Standards - Algebra I

Functions

Interpreting Functions (F-IF)

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

There is no additional scope or clarification information for this standard.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

There is no additional scope or clarification information for this standard.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

i) Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

The function types listed here are the same as those listed in the Algebra I column for standards F-IF.4 and F-IF.6.

Building Functions (F-BF)

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.★
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

i) Tasks have a real-world context.
ii) Tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

i) Identifying the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of k (both positive and negative) is limited to linear and quadratic functions.
ii) Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.
iii) Tasks do not involve recognizing even and odd functions.

The function types listed in note (ii) are the same as those listed in the Algebra I column for standards F-IF.4, F-IF.6, and F-IF.9.

Tennessee's State Mathematics Standards - Algebra I

Linear, Quadratic, and Exponential Models ★ (F–LE)

	Standard	Scope/Clarification
Construct and compare linear, quadratic, and exponential models and solve problems	1. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	*There is no additional scope or clarification information for this standard.*
	2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	i) Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).
	3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	*There is no additional scope or clarification information for this standard.*
Interpret expressions for functions in terms of the situation they model	5. Interpret the parameters in a linear or exponential function in terms of a context.	i) Tasks have a real-world context. ii) Exponential functions are limited to those with domains in the integers.

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Tennessee's State Mathematics Standards - Algebra I

			Standard	Additional scope or clarification information
Statistics and Probability	**Interpreting Categorical and Quantitative Data (S-ID)**	Summarize, represent, and interpret data on a single count or measurement variable	1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	*There is no additional scope or clarification information for this standard.*
			2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	*There is no additional scope or clarification information for this standard.*
			3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	*There is no additional scope or clarification information for this standard.*
		Summarize, represent, and interpret data on two categorical and quantitative variables	5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	*There is no additional scope or clarification information for this standard.*
			6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	For S-ID.6a: i) Tasks have a real-world context. ii) Exponential functions are limited to those with domains in the integers.
		Interpret linear	7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	*There is no additional scope or clarification information for this standard.*
			8. Compute (using technology) and interpret the correlation coefficient of a linear fit.	*There is no additional scope or clarification information for this standard.*
			9. Distinguish between correlation and causation.	*There is no additional scope or clarification information for this standard.*

Major Content	Supporting Content	Additional Content

*Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category, and specific modeling standards appear throughout the high school standards indicated with a star (*). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Tennessee's State Mathematics Standards - Algebra II

Number and Quantity

Domain	Cluster	Standard	Scope and Clarifications
The Real Number System (N-RN)	Extend the properties of exponents to rational exponents	1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	*There is no additional scope or clarification information for this standard.*
		2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	*There is no additional scope or clarification information for this standard.*
Quantities* (N-Q)	Reason quantitatively and use units to solve problems.	2. Define appropriate quantities for the purpose of descriptive modeling.	This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude.
The Complex Number System (N-CN)	Perform arithmetic operations with complex numbers.	1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	*There is no additional scope or clarification information for this standard.*
		2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	*There is no additional scope or clarification information for this standard.*
	Use complex numbers and equations.	7. Solve quadratic equations with real coefficients that have complex solutions.	*There is no additional scope or clarification information for this standard.*

Tennessee's State Mathematics Standards - Algebra II

Domain	Cluster	Standard	Scope and Clarifications
Seeing Structure in Expressions (A-SSE)	**Interpret the structure of expressions**	2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	i) Tasks are limited to polynomial, rational, or exponential expressions. ii) Examples: see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. In the equation $x^2 + 2x + 1 + y^2 = 9$, see an opportunity to rewrite the first three terms as $(x+1)^2$, thus recognizing the equation of a circle with radius 3 and center (-1, 0). See $(x^2 + 4)/(x^2 + 3)$ as $((x^2 + 3) + 1)/(x^2 + 3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.
	Write expressions in equivalent forms to solve problems	3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation. ii) Tasks are limited to exponential expressions with rational or real exponents.
		4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★	*There is no additional scope or clarification information for this standard.*
Arithmetic with Polynomials and Rational Expressions (A-APR)	**Understand the relationship between zeros and factors of Polynomials**	2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	*There is no additional scope or clarification information for this standard.*
		3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	i) Tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x + 1)$.
	Use polynomial identities to solve problems	4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	*There is no additional scope or clarification information for this standard.*
	Rewrite rational expressions	6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	*There is no additional scope or clarification information for this standard.*

Tennessee's State Mathematics Standards - Algebra II

Algebra

Creating Equations★ (A-CED)

Category	Standard	Scope & Clarification
Create equations that describe numbers or relationships	1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	i) Tasks are limited to exponential equations with rational or real exponents and rational functions. ii) Tasks have a real-world context.

Reasoning with Equations and Inequalities (A-REI)

Category	Standard	Scope & Clarification
Understand solving equations as a process of reasoning and explain the reasoning	1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	i) Tasks are limited to simple rational or radical equations.
	2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	*There is no additional scope or clarification information for this standard.*
Solve equations and inequalities in one variable	4. Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	i) In the case of equations that have roots with nonzero imaginary parts, students write the solutions as $a \pm bi$ for real numbers a and b.
Solve systems of equations	6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	i) Tasks are limited to 3x3 systems.
	7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*	*There is no additional scope or clarification information for this standard.*
Represent and solve equations and inequalities graphically	11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	i) Tasks may involve any of the function types mentioned in the standard.

Tennessee's State Mathematics Standards - Algebra II

Functions

Interpreting Functions (F-IF)

Cluster	Standard	Scope & Clarifications
Understand the concept of a function and use function notation	3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*	i) This standard is Supporting work in Algebra II. This standard should support the Major work in F-BF.2 for coherence.
Interpret functions that arise in applications in terms of the context	4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★	i) Tasks have a real-world context ii) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. *Compare note (ii) with standard F-IF.7.* *The function types listed here are the same as those listed in the Algebra II column for standards F-IF.6 and F-IF.9.*
	6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★	i) Tasks have a real-world context. ii) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. *The function types listed here are the same as those listed in the Algebra II column for standards F-IF.4 and F-IF.9.*
Analyze functions using	7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★	
	c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	*There is no additional scope or clarification information for this standard.*
	e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
	8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	*There is no additional scope or clarification information for this standard.*
	b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.*	
	9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	i) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. *The function types listed here are the same as those listed in the Algebra II column for standards F-IF.4 and F-IF.6.*

Tennessee's State Mathematics Standards - Algebra II

Functions

		Standard	Scope & Clarifications
Building Functions (F-BF)	Build a function that models a relationship between two quantities	1. Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	For F-BF.1a: i) Tasks have a real-world context ii) Tasks may involve linear functions, quadratic functions, and exponential functions.
	Build new functions from existing functions	2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★	*There is no additional scope or clarification information for this standard.*
		3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	i) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions ii) Tasks may involve recognizing even and odd functions. *The function types listed in note (i) are the same as those listed in the Algebra II column for standards F-IF.4, F-IF.6, and F-IF.9.*
		4. Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) =2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*	*There is no additional scope or clarification information for this standard.*
Linear, Quadratic, and Exponential Models★ (F-LE)	Construct and compare linear, quadratic, and exponential models and solve problems	2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	i) Tasks will include solving multi-step problems by constructing linear and exponential functions.
		4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	*There is no additional scope or clarification information for this standard.*
	Interpret expressions for functions in terms of the situation they model	5. Interpret the parameters in a linear or exponential function in terms of a context.	i) Tasks have a real-world context. ii) Tasks are limited to exponential functions with domains not in the integers.

Tennessee's State Mathematics Standards - Algebra II

				Scope & Clarification
Functions	**Trigonometric Functions (F-TF)**	Extend the domain of trigonometric functions using the unit circle	1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	*There is no additional scope or clarification information for this standard.*
			2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	*There is no additional scope or clarification information for this standard.*
		Model periodic phenomena with trigonometric functions	5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★	*There is no additional scope or clarification information for this standard.*
		Prove and apply trigonometric identities	8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	*There is no additional scope or clarification information for this standard.*
Geometry	**Expressing Geometric Properties with Equations (G-GPE)**	Translate between the geometric description and the equation for a conic section	2. Derive the equation of a parabola given a focus and directrix.	*There is no additional scope or clarification information for this standard.*

Tennessee's State Mathematics Standards - Algebra II

Statistics and Probability

Interpreting Categorical and Quantitative Data (S-ID)

Summarize, represent, and interpret data on a single count or measurement variable

Standard	Scope & Clarification
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	*There is no additional scope or clarification information for this standard.*

Summarize, represent, and interpret data on two categorical and quantitative variables

Standard	Scope & Clarification
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*	i) Tasks have a real-world context. ii) Tasks are limited to exponential functions with domains not in the integers and trigonometric functions.

Making Inferences and Justifying Conclusions (S-IC)

Understand and evaluate random processes underlying statistical experiments

Standard	Scope & Clarification
1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	*There is no additional scope or clarification information for this standard.*
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	*There is no additional scope or clarification information for this standard.*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Standard	Scope & Clarification
3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	*There is no additional scope or clarification information for this standard.*
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	*There is no additional scope or clarification information for this standard.*
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	*There is no additional scope or clarification information for this standard.*
6. Evaluate reports based on data.	*There is no additional scope or clarification information for this standard.*

Tennessee's State Mathematics Standards - Algebra II

Statistics and Probability

Conditional Probability and the Rules of Probability (S-CP)

Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

There is no additional scope or clarification information for this standard.

2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

There is no additional scope or clarification information for this standard.

3. Understand the conditional probability of A given B as $P(A$ and $B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

There is no additional scope or clarification information for this standard.

4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

There is no additional scope or clarification information for this standard.

5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

There is no additional scope or clarification information for this standard.

Use the rules of probability to compute probabilities of compound events in a uniform probability model

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

There is no additional scope or clarification information for this standard.

7. Apply the Addition Rule, $P(A$ or $B) = P(A) + P(B) - P(A$ and $B)$, and interpret the answer in terms of the model.

There is no additional scope or clarification information for this standard.

Major Content	Supporting Content	Additional Content

*Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category, and specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Geometry

Domain	Cluster	Standard
Geometry **Congruence** **(G-CO)**	**Experiment with transformations in the plane**	1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
		2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
		3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
		4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
		5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
	Understand congruence in terms of rigid motions	6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
		7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
		8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Geometry

TNCore

Geometry		
Congruence (G-CO)	**Prove geometric theorems**	9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
		10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
		11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*
	Make geometric constructions	12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
		13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

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Geometry			
Similarity, Right Triangles, and Trigonometry (G-SRT)	**Understand similarity in terms of similarity transformations**	1. Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	
		2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
		3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
	Prove theorems involving similarity	4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	
		5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
	Define trigonometric ratios and solve problems involving right triangles	6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
		7. Explain and use the relationship between the sine and cosine of complementary angles.	
		8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★	

Geometry

Geometry			
Circles (G-C)	Understand and apply theorems about circles	1. Prove that all circles are similar.	
		2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	
		3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	
	Find arc lengths and areas of sectors of circles	5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	
Expressing Geometric Properties with Equations (G-GPE)	Translate between the geometric description and the equation for a conic section	1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	
	Use coordinates to prove simple geometric theorems algebraically	4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.*	
		5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	
		6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
		7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★	

Geometry

Geometry			
Geometric Measurement and Dimension (G-GMD)	Explain volume formulas and use them to solve problems	1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	
		3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★	
	Visualize relationships between two-dimensional and three-dimensional objects	4. Identify the shapes of two-dimensional cross-sections of three dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	
Modeling with Geometry (G-MG)	Apply geometric concepts in modeling situations	1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★	
		2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★	
		3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★	

Major Content	Supporting Content	Additional Content

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TNCore Tools

Appendix C
Structure and Routines of a Lesson

The Structure and Routines of a Lesson

Set Up of the Task	**MONITOR:** Teacher selects examples for the Share, Discuss, and Analyze phase based on: • Different solution paths to the same task • Different representations • Errors • Misconceptions
The Explore Phase/ Private Work Time Generate Solutions **The Explore Phase/Small Group Problem Solving** 1. Generate and Compare Solutions 2. Assess and Advance Student Learning	**SHARE:** Students explain their methods, repeat others' ideas, put ideas into their own words, add on to ideas, and ask for clarification. *REPEAT THE CYCLE FOR EACH SOLUTION PATH.* **COMPARE:** Students discuss similarities and differences between solution paths.
Share, Discuss, and Analyze Phase of the Lesson 1. Share and Model 2. Compare Solutions 3. Focus the Discussion on Key Mathematical Ideas 4. Engage in a Quick Write	**FOCUS:** Discuss the meaning of mathematical ideas in each representation. **REFLECT:** Engage students in a Quick Write or a discussion of the process.

Appendix D
Math Accountable Talk®
Academic Discussion

Accountable Talk® Features and Indicators

Accountability to the Learning Community
- Active participation in classroom talk
- Listen attentively
- Elaborate and build on each other's ideas
- Work to clarify or expand a proposition

Accountability to Knowledge
- Specific and accurate knowledge
- Appropriate evidence for claims and arguments
- Commitment to getting it right

Accountability to Rigorous Thinking
- Synthesize several sources of information
- Construct explanations and test understanding of concepts
- Formulate conjectures and hypotheses
- Employ generally accepted standards of reasoning
- Challenge the quality of evidence and reasoning

Accountable Talk® Moves

Talk Move	Function	Example
To Ensure Purposeful, Coherent, and Productive Group Discussion		
Marking	Direct attention to the value and importance of a student's contribution.	That's an important point.
Challenging	Redirect a question back to the students or use students' contributions as a source for further challenge or query.	Let me challenge you: Is that always true?
Revoicing	Align a student's explanation with content or connect two or more contributions with the goal of advancing the discussion of the content.	S: 4 + 4 + 4. You said three groups of four.
Recapping	Make public in a concise, coherent form, the group's achievement at creating a shared understanding of the phenomenon under discussion.	Let me put these ideas all together. What have we discovered?

Accountable Talk® Moves

Talk Move	Function	Example
To Ensure Purposeful, Coherent, and Productive Group Discussion		
Marking	Direct **attention** to the value and importance of a student's contribution.	It is important to say describe to compare the size of the pieces and then to look at how many pieces of that size.
Challenging	Redirect a question back to the students or use students' contributions as a source for further challenge or query.	Let me challenge you: Is that always true?
Revoicing	Align a student's explanation with content or connect two or more contributions with the goal of advancing the discussion of the content.	You said 3, yes there are three columns and each column is 1/3 of the whole
Recapping	Make public in a concise, coherent form, the group's achievement at creating a shared understanding of the phenomenon under discussion.	Let me put these ideas all together. What have we discovered?
To Support Accountability to Community		
Keeping the Channels Open	Ensure that students can hear each other, and remind them that they must hear what others have said.	Say that again and louder. Can someone repeat what was just said?
Keeping Everyone Together	Ensure that everyone not only heard, but also understood, what a speaker said.	Can someone add on to what was said? Did everyone hear that?
Linking Contributions	Make explicit the relationship between a new contribution and what has gone before.	Does anyone have a similar idea? Do you agree or disagree with what was said? Your idea sounds similar to his idea.
Verifying and Clarifying	Revoice a student's contribution, thereby helping both speakers and listeners to engage more profitably in the conversation.	So are you saying..? Can you say more? Who understood what was said?
To Support Accountability to Knowledge		
Pressing for Accuracy	Hold students accountable for the accuracy, credibility, and clarity of their contributions.	Why does that happen? Someone give me the term for that.
Building on Prior Knowledge	Tie a current contribution back to knowledge accumulated by the class at a previous time.	What have we learned in the past that links with this?
To Support Accountability to Rigorous Thinking		
Pressing for Reasoning	Elicit evidence to establish what contribution a student's utterance is intended to make within the group's larger enterprise.	Say why this works. What does this mean? Who can make a claim and then tell us what their claim means?
Expanding Reasoning	Open up extra time and space in the conversation for student reasoning.	Does the idea work if I change the context? Use bigger numbers?

Appendix E
Task Analysis Guide

The Mathematical Task Analysis Guide

Lower-Level Demands
Memorization Tasks

- Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.

Procedures Without Connections Tasks

- Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers rather than developing mathematical understanding.
- Require no explanations, or explanations that focus solely on describing the procedure that was used.

Higher-Level Demands
Procedures With Connections Tasks

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

Doing Mathematics Tasks

- Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).
- Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships.
- Demands self-monitoring or self-regulation of one's own cognitive processes.
- Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Mathematics Teaching in the Middle School. Also in: Stein, Smith, Henningsen, & Silver (2000). Implementing standards-based mathematics instruction: A casebook for professional development, p. 16. New York: Teachers College Press.

Appendix F
Strategies for Modifying Textbook Tasks

Strategies for Modifying Textbook Tasks

Compare your list of task modifications with the list of task modification strategies identified by others. How is your list similar? Different?

- Ask students to create real-world stories for "naked number" problems.

- Include a prompt that asks students to represent the information another way (with a picture, in a table, a graph, an equation, with a context).

- Include a prompt that requires students to make a generalization.

- Use a task "out of sequence" before students have memorized a rule or have practiced a procedure that can be routinely applied.

- Eliminate components of the task that provide too much scaffolding.

- Adapt a task so as to provide more opportunities for students to think and reason – let students figure things out for themselves.

- Create a prompt that asks students to write about the meaning of the mathematics concept.

- Add a prompt that asks students to make note of a pattern or to make a mathematical conjecture and to test their conjecture.

- Include a prompt that requires students to compare solution paths or mathematical relationships and write about the relationship between strategies or concepts.

- Select numbers carefully so students are more inclined to note relationships between quantities (e.g., two tables can be used to think about the solutions to the four, six or eight tables).

TNReady

Appendix G
TNBlueprints

TNReady Core Math I Blueprint

Clusters on Part I	# of items in Part I	% of Part I	Additional Clusters on Part II (All Part I Clusters will also be assessed on Part II)	# of items in Part II	% of Part II	% of test
Structure and interpretation of expressions and quantities • Use units to solve problems • Interpret the structure of expressions • Write expressions in equivalent forms	5-7	21-23%	No additional clusters	4-6	11-13%	15-17%
Create equations that describe numbers or relationships	4-6	16-18%	No additional clusters	2-4	7-9%	11-13%
Reason with equations and inequalities • Solve equations and inequalities	1-3	4-6%	**Reason with equations and inequalities** • Solve systems of equations • Represent and solve equations and inequalities graphically	6-8	17-19%	11-13%
Interpreting and building functions • Understand the concept of a function and use function notation • Interpret functions that arise in applications in terms of the context • Build a function that models a relationship between two quantities	4-6	19-21%	**Interpreting and building functions** • Analyze functions using different representations	8-10	21-23%	20-22%
Construct and compare linear, quadratic and exponential functions • Construct and compare linear, quadratic, and exponential models and solve problems	2-4	11-13%	**Construct and compare linear, quadratic and exponential functions** • Interpret expressions for functions in terms of the situation they model	2-4	7-9%	8-10%
Geometry: Congruence • Experiment with transformations in the plane • Understand congruence in terms of rigid motions • Prove geometric theorems	5-7	21-23%	No additional clusters	4-6	12-14%	15-17%
No content from these clusters will be assessed on Part I	0	0%	**Interpreting categorical and quantitative data** • Summarize, represent, and interpret data on a single count or measurement variable • Summarize, represent, and interpret data on two categorical and quantitative variables • Interpret linear models	6-8	17-19%	9-11%
Total	21-33	100%	Total	32-46	100%	100%

Part I – Calculator Allowed

Cluster	Standards		# of Items
Structure and interpretation of expressions and quantities	N-Q – Reason quantitatively and use units to solve problems	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	5-7
		Define appropriate quantities for the purpose of descriptive modeling.	
		Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	
	A-SSE.A – Interpret the structure of expressions	Interpret expressions that represent a quantity in terms of its context. a.　Interpret parts of an expression, such as terms, factors, and coefficients. b.　Interpret complicated expressions by viewing one or more of their parts as a single entity.	
	A-SSE.B – Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c.　Use the properties of exponents to transform expressions for exponential functions.	
Create equations that describe numbers or relationships	A-CED – Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	4-6
		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
		Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	
		Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	
Reason with equations and inequalities	A-REI.B – Solve equations and inequalities in one variable	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	1-3
Interpreting and building functions	F-IF.A – Understand the concept of a function and use function notation	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	4-6
		Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
		Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	
	F-IF.B – Interpret functions that arise in applications in terms of context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	

	F-BF.A – Build a function that models a relationship between two quantities	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
		Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
		Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	
Construct and compare linear, quadratic and exponential functions	F-LE.A – Construct and compare linear, quadratic, and exponential models and solve problems	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	2-4
		Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
		Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
Geometry: Congruence	G-CO.A – Experiment with transformations in the plane	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	5-7
		Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
		Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
		Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
		Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
	G.CO.B – Understand congruence in terms of rigid motions	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	
		Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	

	G.CO.C – Prove geometric theorems	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
		Prove theorems about lines and angles.	
		Prove theorems about triangles.	
		Prove theorems about parallelograms.	

Part II – Calculator and Non-Calculator Portions

Cluster	Standards		# of Items
Structure and operations with expressions and quantities	N-Q – Reason quantitatively and use units to solve problems	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	4-6
		Define appropriate quantities for the purpose of descriptive modeling.	
		Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	
	A-SSE.A – Interpret the structure of expressions	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	
	A-SSE.B – Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions.	
Create equations that describe numbers or relationships	A-CED – Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	2-4
		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
		Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	
		Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	
Reason with equations and inequalities	A.REI.B – Solve equations and inequalities in one variable	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	6-8
	A-REI.C – Solve systems of equations	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
		Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
	A-REI.D – Represent and solve equations and inequalities graphically	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
		Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	

		Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	
Interpreting and building functions	F-IF.A – Understand the concept of a function and use function notation	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	8-10
		Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
		Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	
	F-IF.B – Interpret functions that arise in applications in terms of context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	
		Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
	F-IF.C – Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F-BF.A – Build a function that models a relationship between two quantities	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
		Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	
Construct and compare linear, quadratic and exponential functions	F-LE.A – Construct and compare linear, quadratic, and exponential models and solve problems	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	2-4

		Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
		Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
	F-LE.B – Interpret expressions for functions in terms of the situation they model	Interpret the parameters in a linear or exponential function in terms of a context.	
Geometry: Congruence	G-CO.A – Experiment with transformations in the plane	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	4-6
		Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
		Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
		Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
		Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
	G.CO.B – Understand congruence in terms of rigid motions	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	
		Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
		Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
	G.CO.C – Prove geometric theorems	Prove theorems about lines and angles.	
		Prove theorems about triangles.	
		Prove theorems about parallelograms.	

		Represent data with plots on the real number line (dot plots, histograms, and box plots).	
Interpreting categorical and quantitative data	S-ID.A – Summarize, represent, and interpret data on a single count or measurement variable	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	6-8
		Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	
	S-ID.B – Summarize, represent, and interpret data on two categorical and quantitative variables	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	
		Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. b. Fit a linear function for a scatter plot that suggests a linear association.	
	S-ID.C – Interpret linear models	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	
		Compute (using technology) and interpret the correlation coefficient of a linear fit.	
		Distinguish between correlation and causation.	

Overall Blueprint (Includes Part I and Part II)

Cluster	Standards		# of Items
Structure and operations with expressions and quantities	N-Q – Reason quantitatively and use units to solve problems	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	9-13
		Define appropriate quantities for the purpose of descriptive modeling.	
		Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	
	A-SSE.A – Interpret the structure of expressions	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	
	A-SSE.B – Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions.	
Create equations that describe numbers or relationships	A-CED – Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	6-10
		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
		Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	
		Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	
Reason with equations and inequalities	A-REI.B – Solve equations and inequalities in one variable	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	7-11
	A-REI.C – Solve systems of equations	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
		Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
	A-REI.D – Represent and solve equations and inequalities graphically	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	

		Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	
		Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	
Interpreting and building functions	F-IF.A – Understand the concept of a function and use function notation	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	12-16
		Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
		Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	
	F-IF.B – Interpret functions that arise in applications in terms of context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	
		Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
	F-IF.C – Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F-BF.A – Build a function that models a relationship between two quantities	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
		Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	
Construct and compare linear,	F-LE.A – Construct and compare linear,	Distinguish between situations that can be modeled with linear functions and with exponential functions. b. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. c. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. d. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	4-8

quadratic and exponential functions	quadratic, and exponential models and solve problems	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
		Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
	F-LE.B – Interpret expressions for functions in terms of the situation they model	Interpret the parameters in a linear or exponential function in terms of a context.	
Geometry: Congruence	G-CO.A – Experiment with transformations in the plane	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	9-13
		Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
		Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
		Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
		Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
	G-CO.B – Understand congruence in terms of rigid motions	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	
		Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
		Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
	G-CO.C – Prove geometric theorems	Prove theorems about lines and angles.	
		Prove theorems about triangles.	
		Prove theorems about parallelograms.	
Interpreting categorical and	S-ID.A – Summarize, represent, and	Represent data with plots on the real number line (dot plots, histograms, and box plots).	6-8

quantitative data	interpret data on a single count or measurement variable	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	
		Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	
	S-ID.B – Summarize, represent, and interpret data on two categorical and quantitative variables	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	
		Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. *a.* Fit a function to the data; use functions fitted to data to solve problems in the context of the data. b. Fit a linear function for a scatter plot that suggests a linear association.	
	S-ID.C – Interpret linear models	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	
		Compute (using technology) and interpret the correlation coefficient of a linear fit.	
		Distinguish between correlation and causation.	

TNReady Core Math II Blueprint

Clusters on Part I	# of items	% of Part I	Additional Clusters on Part II (All Part I Clusters will also be assessed on Part II)	# of items	% of Part II	% of test
Number Systems: Real and Complex • Extend the properties of exponents to rational exponents • Use properties of rational and irrational numbers • Perform arithmetic operations with complex numbers • Use complex numbers in polynomial identities and equations	2-4	9-11%	No additional clusters	2-4	6-8%	8-10%
Structure and Operations with Expressions • Use units to solve problems • Interpret the structure of expressions • Write expressions in equivalent forms • Perform arithmetic operations on polynomials	4-6	17-19%	No additional clusters	5-7	14-16%	15-17%
Creating Equations • Create equations that describe numbers or relationships	3-5	14–16%	No additional clusters	2-4	6-8%	9-11%
Reasoning with Equations and Inequalities • Understand solving equations as a process of reasoning and explain the reasoning • Solve equations in one variable	2-4	12-14%	**Reasoning with Equations and Inequalities** • Solve systems of equations	4-6	12-14%	11-13%
Interpreting and Building Functions • Interpret functions that arise in applications in terms of the context • Analyze functions using different representations • Build a function that models a relationship between two quantities	4-6	19-21%	**Interpreting and Building Functions** • Build new functions from existing functions	8-10	22-24%	20-22%

Geometry: Similarity, Right Triangles, Trigonometry, and Dimension	6-8	24-26%	No additional clusters	7-9	19-21%	21-23%
• Understand similarity in terms of similarity transformations • Prove theorems involving similarity • Define trigonometric ratios and solve problems involving right triangles • Explain volume formulas and use them to solve problems						
No content from these clusters will be assessed on Part I	0	0%	Interpreting Data and Understanding Probability • Summarize, represent, and interpret data on two categorical and quantitative variables • Understand independence and conditional probability and use them to interpret data • Use the rules of probability to compute probabilities of compound events in a uniform probability model	5-7	14-16%	8-10%
Total	21-33	100%	Total	33-47	100%	100%

Cluster	Standards		# of Items
Number System: Real and Complex	N-RN.A-Extend the properties of exponents to rational exponents	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	2-4
		Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
	N-RN.B-Use properties of rational and irrational numbers.	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	
	N-CN.A-Perform arithmetic operations with complex numbers	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	
		Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
	N-CN.B-Use complex numbers in polynomial identities and equations.	Solve quadratic equations with real coefficients that have complex solutions	
Structure and Operations with Expressions	N-Q.A – Reason quantitatively and use units to solve problems	Define appropriate quantities for the purpose of descriptive modeling.	4-6
	A-SSE.A – Interpret the structure of expressions	Interpret expressions that represent a quantity in terms of its context.★ b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	
		Use the structure of an expression to identify ways to rewrite it.	
	A-SSE.B-Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	
	A-APR.A-Perform arithmetic operations on polynomials	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
Creating Equations	A-CED.A-Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	3-5
		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
		Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	

Reasoning with Equations and Inequalities	A.REI.A - Understand solving equations as a process of reasoning and explain the reasoning	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	2-4
	A-REI.B- Solve equations and inequalities in one variable	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2$ = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	
Interpreting and Building Functions	F.IF.B - Interpret functions that arise in applications in terms of the context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	4-6
		Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
	F.IF.C - Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
		Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F.BF.A - Build a function that models a relationship between two quantities	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations.	

Geometry: Similarity, Right Triangles, Trigonometry and Dimensions	G.SRT.A- Understand similarity in terms of similarity transformation	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	6-8
		Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
		Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
	G.SRT.B- Prove theorems involving similarity	Prove theorems about triangles.	
		Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
	G.SRT.C- Define trigonometric ratios and solve problems involving right triangles	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
		Explain and use the relationship between the sine and cosine of complementary angles.	
		Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	
	G.GMD.A- Explain volume formulas and use them to solve problems	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	
		Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	

Part II – Calculator and Non-Calculator Portions

Cluster	Standards		# of Items
Number System: Real and Complex	N-RN.A-Extend the properties of exponents to rational exponents	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	2-4
		Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
	N-RN.B-Use properties of rational and irrational numbers.	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	
	N-CN.A-Perform arithmetic operations with complex numbers	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	
		Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
	N-CN.B - Use complex numbers in polynomial identities and equations.	Solve quadratic equations with real coefficients that have complex solutions	
Structure and Operations with Expressions	N-Q.A – Reason quantitatively and use units to solve problems	Define appropriate quantities for the purpose of descriptive modeling.	5-7
	A-SSE.A – Interpret the structure of expressions	Interpret expressions that represent a quantity in terms of its context. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	
		Use the structure of an expression to identify ways to rewrite it.	
	A-SSE.B-Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	
	A-APR- Perform arithmetic operations on polynomials	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
Creating Equations	A-CED-Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	2-4
		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
		Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	

Cluster	Standards		# of Items
Reasoning with Equations and Inequalities	A.REI.A - Understand solving equations as a process of reasoning and explain the reasoning	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	4-6
	A-REI.B- Solve equations and inequalities in one variable	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2$ = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	
	A.REI.C- Solve systems of equations	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	
Interpreting and Building Functions	F.IF.B - Interpret functions that arise in applications in terms of the context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	8-10
		Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
	F.IF.C- Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
		Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F.BF.A- Build a function that models a relationship between two	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations.	

Cluster	Standards		# of Items
	quantities		
	F.BF.B- Build new functions from existing functions	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	
Geometry: Similarity, Right Triangles, Trigonometry and Dimensions	G.SRT.A- Understand similarity in terms of similarity transformation	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	7-9
		Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
		Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
	G.SRT.B- Prove theorems involving similarity	Prove theorems about triangles.	
		Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
	G.SRT.C- Define trigonometric ratios and solve problems involving right triangles	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
		Explain and use the relationship between the sine and cosine of complementary angles.	
		Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	
	G.GMD.A- Explain volume formulas and use them to solve problems	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	
		Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	
Interpreting Data and Understanding Probability	S.ID.A- Summarize, represent, and interpret data on two categorical and quantitative variables	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. b. Informally assess the fit of a function by plotting and analyzing residuals.	5-7
	S.CP.A- Understand independence and conditional probability and use them to interpret data	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	
		Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	

Cluster	Standards		# of Items
		Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	
		Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	
		Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	
	S.CP.B- Use the rules of probability to compute probabilities of compound events in a uniform probability model	Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.	
		Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.	

Overall Blueprint (Includes Part I and Part II)

Cluster	Standards		# of Items
Number System: Real and Complex	N-RN.A-Extend the properties of exponents to rational exponents	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	4-8
		Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
	N-RN.B-Use properties of rational and irrational numbers.	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	
	N-CN.A-Perform arithmetic operations with complex numbers	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	
		Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
	N-CN.B- Use complex numbers in polynomial identities and equations.	Solve quadratic equations with real coefficients that have complex solutions	
Structure and Operations with Expressions	N-Q.A – Reason quantitatively and use units to solve problems	Define appropriate quantities for the purpose of descriptive modeling.	9-13
	A-SSE.A – Interpret the structure of expressions	Interpret expressions that represent a quantity in terms of its context. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	
		Use the structure of an expression to identify ways to rewrite it.	
	A-SSE.B-Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	
	A-APR.A-Perform arithmetic operations on polynomials	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
Creating Equations	A-CED.A-Create equations that describe numbers or	Create equations and inequalities in one variable and use them to solve problems.	5-9
		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	

Cluster	Standards		# of Items
	relationships	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	
Reasoning with Equations and Inequalities	A.REI.A - Understand solving equations as a process of reasoning and explain the reasoning	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	6-10
	A-REI.B- Solve equations and inequalities in one variable	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2$ = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	
	A.REI.C- Solve systems of equations	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	
Interpreting and Building Functions	F.IF.B - Interpret functions that arise in applications in terms of the context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	12-16
		Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
	F.IF.C- Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
		Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F.BF.A- Build a function that	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation	

Cluster	Standards		# of Items
	models a relationship between two quantities	from a context. b. Combine standard function types using arithmetic operations.	
	F.BF.B - Build new functions from existing functions	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	
Geometry: Similarity, Right Triangles, Trigonometry and Dimensions	G.SRT.A- Understand similarity in terms of similarity transformation	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	13-17
		Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
		Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
	G.SRT.B- Prove theorems involving similarity	Prove theorems about triangles.	
		Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
	G.SRT.C- Define trigonometric ratios and solve problems involving right triangles	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
		Explain and use the relationship between the sine and cosine of complementary angles.	
		Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	
	G.GMD.A- Explain volume formulas and use them to solve problems	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	
		Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	
Interpreting Data and Understanding Probability	S.ID.A- Summarize, represent, and interpret data on two categorical and quantitative variables	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. b. Informally assess the fit of a function by plotting and analyzing residuals.	5-7
	S.CP.A- Understand independence and conditional probability and	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	
		Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this	

Cluster	Standards		# of Items
	use them to interpret data	characterization to determine if they are independent.	
		Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	
		Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	
		Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	
	S.CP.B- Use the rules of probability to compute probabilities of compound events in a uniform probability model	Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.	
		Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.	

TNReady Core Math III Blueprint

Clusters on Part I	# of items	% of Part I	Additional Clusters on Part II (All Part I Clusters will also be assessed on Part II)	# of items	% of Part II	% of test
Structure and Operations with Expressions • Reason quantitatively and use units to solve problems • Interpret the structure of expressions • Write expressions in equivalent forms • Understand the relationship between zeros and factors of polynomials • Use polynomial identities to solve problems • Rewrite rational expressions	6-8	24-26%	No additional clusters	5-7	14-16%	18-20%
Creating and Reasoning with Equations and Inequalities • Create equations that describe numbers or relationships • Understand solving equations as a process of reasoning and explain the reasoning • Represent and solve equations and inequalities graphically	4-6	19-21%	No additional clusters	3-5	9-11%	13-15%
Interpreting and Building Functions • Interpret functions that arise in applications in terms of the context • Analyze functions using different representations • Build new functions from existing functions	4-6	17-19%	No additional clusters	4-6	12-14%	14-16%
Linear, Quadratic, Exponential and Trigonometric Functions • Construct and compare linear, quadratic, and exponential models and solve problems	1-3	4-6%	**Linear, Quadratic, Exponential and Trigonometric Functions** • Extend the domain of trigonometric functions using the unit circle • Model periodic phenomena with trigonometric functions • Prove and apply trigonometric identities	5-7	14-16%	10-12%

Geometry: Congruence and Constructions • Make geometric constructions • Understand and apply theorems about circles • Find arc lengths and areas of sectors of circles	3-5	14-16%	No additional clusters	2-4	7-9%	9-12%
Geometry: Properties, Dimension and Modeling • Translate between the geometric description and the equation for a conic section • Use coordinates to prove simple geometric theorems algebraically • Apply geometric concepts in modeling situations	4-6	17-19%	Geometry: Properties, Dimension and Modeling • Visualize relationships between two-dimensional and three-dimensional objects	4-6	12-14%	14-16%
No content from these clusters will be assessed on Part I	0	0%	Interpreting Data, Making Inferences and Justifying Conclusions • Summarize, represent, and interpret data on a single count or measurement variable • Summarize, represent, and interpret data on two categorical and quantitative variables • Understand and evaluate random processes underlying statistical experiments • Make inferences and justify conclusions from sample surveys, experiment, and observational studies	9-11	24-26%	14-16%
Total	22-34	100%	Total	32-46	100%	100%

Part I – Calculator Allowed

Cluster	Standards		# of Items
Structure and Operation with Expressions	N-Q.A – Reason quantitatively and use units to solve problems.	Define appropriate quantities for the purpose of descriptive modeling.	6-8
	A-SSE.A – Interpret the structure of expressions	Use the structure of an expression to identify ways to rewrite it.	
	A-SSE.B –Write expressions in equivalent forms to solve problems	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.	
	A-APR.B – Understand the relationship between zeros and factors of polynomials	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.	
		Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
	A-APR.C – Use polynomial identities to solve problems	Prove polynomial identities and use them to describe numerical relationships.	
	A-APR.D – Rewrite rational expressions	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.	
Creating and Reasoning with Equations and Inequalities	A-CED.A – Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	4-6
		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
	A-REI.A – Understand solving equations as a process of reasoning and explain the reasoning	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
		Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
	A-REI.D – Represent and solve equations and inequalities graphically	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	

Cluster	Standards		# of Items
Interpreting and Building Functions	F-IF.B – Interpret functions that arise in applications in terms of the context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	4-6
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
	F-IF.C – Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F-BF.B – Build new functions from existing functions	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	
		Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.	
Linear, Quadratic, Exponential, and Trigonometric Functions	F-LE.A – Construct and compare linear, quadratic, and exponential model sand solve problems	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	1-3
Geometry: Congruence and Constructions	G-CO.D – Make geometric constructions	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).	3-5
		Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	
	G-C.A – Understand and apply theorems about circles	Prove that all circles are similar.	
		Identify and describe relationships among inscribed angles, radii, and chords.	
		Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	
	G-C.B – Find arc lengths and areas of sectors of circles	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	

Cluster	Standards	168	# of Items
Geometry: Properties, Dimension, and Modeling	G-GPE.A – Translate between the geometric description and the equation for a conic section	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	4-6
		Derive the equation of a parabola given a focus and directrix.	
	G-GPE.B – Use coordinates to prove simple geometric theorems algebraically	Use coordinates to prove simple geometric theorems algebraically.	
		Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	
		Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
		Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	
	G-MG.A – Apply geometric concepts in modeling situations	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	
		Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	
		Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	

Part II – Calculator and Non-Calculator Portions

Cluster	Standards		# of Items
Structure and Operation with Expressions	N-Q.A – Reason quantitatively and use units to solve problems.	Define appropriate quantities for the purpose of descriptive modeling.	5-7
	A-SSE.A – Interpret the structure of expressions	Use the structure of an expression to identify ways to rewrite it.	
	A-SSE.B –Write expressions in equivalent forms to solve problems	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.	
	A-APR.B – Understand the relationship between zeros and factors of polynomials	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	
		Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
	A-APR.C – Use polynomial identities to solve problems	Prove polynomial identities and use them to describe numerical relationships.	
	A-APR.D – Rewrite rational expressions	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.	
Creating and Reasoning with Equations and Inequalities	A-CED.A – Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	3-5
		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
	A-REI.A – Understand solving equations as a process of reasoning and explain the reasoning	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
		Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
	A-REI.D – Represent and solve equations and inequalities graphically	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	

Cluster	Standards		# of Items
Interpreting and Building Functions	F-IF.B – Interpret functions that arise in applications in terms of the context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	4-6
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
	F-IF.C – Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F-BF.B – Build new functions from existing functions	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	
		Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.	
Linear, Quadratic, Exponential, and Trigonometric Functions	F-LE.A – Construct and compare linear, quadratic, and exponential model sand solve problems	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	5-7
	F-TF.A – Extend the domain of trigonometric functions using the unit circle	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	
		Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	
	F-TF.B – Model periodic phenomena with trigonometric functions	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	
	F-TF-C – Prove and apply trigonometric identities	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	

Cluster	Standards		# of Items
Geometry: Congruence and Constructions	G-CO.D – Make geometric constructions	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).	2-4
		Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	
	G-C.A – Understand and apply theorems about circles	Prove that all circles are similar.	
		Identify and describe relationships among inscribed angles, radii, and chords.	
		Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	
	G-C.B – Find arc lengths and areas of sectors of circles	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	
Geometry: Properties, Dimension, and Modeling	G-GPE.A – Translate between the geometric description and the equation for a conic section	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	4-6
		Derive the equation of a parabola given a focus and directrix.	
	G-GPE.B – Use coordinates to prove simple geometric theorems algebraically	Use coordinates to prove simple geometric theorems algebraically.	
		Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	
		Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
		Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	
	G-MG.A – Apply geometric concepts in modeling situations	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	
		Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	
		Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	
	G-GMD – Visualize relationships between two-dimensional and three-dimensional objects	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	

Cluster	Standards		# of Items
Interpreting Data, Making Inferences and Justifying Conclusions	S-ID.A – Summarize, represent, and interpret data on a single count or measurement variable	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	9-11
	S-ID.B – Summarize, represent, and interpret data on two categorical and quantitative variables	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. b. Informally assess the fit of a function by plotting and analyzing residuals.	
	S-IC.A – Understand and evaluate random processes underlying statistical experiments	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	
		Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.	
	S-IC.B – Make inferences and justify conclusions from sample surveys, and observational studies	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	
		Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	
		Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	
		Evaluate reports based on data.	

Overall Blueprint (Includes Part I and Part II)

Cluster	Standards		# of Items
Structure and Operation with Expressions	N-Q.A – Reason quantitatively and use units to solve problems.	Define appropriate quantities for the purpose of descriptive modeling.	11-15
	A-SSE.A – Interpret the structure of expressions	Use the structure of an expression to identify ways to rewrite it.	
	A-SSE.B –Write expressions in equivalent forms to solve problems	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.	
	A-APR.B – Understand the relationship between zeros and factors of polynomials	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	
		Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
	A-APR.C – Use polynomial identities to solve problems	Prove polynomial identities and use them to describe numerical relationships.	
	A-APR.D – Rewrite rational expressions	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.	
Creating and Reasoning with Equations and Inequalities	A-CED.A – Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	7-11
		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
	A-REI.A – Understand solving equations as a process of reasoning and explain the reasoning	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
		Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
	A-REI.D – Represent and solve equations and inequalities graphically	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	

Cluster	Standards		# of Items
Interpreting and Building Functions	F-IF.B – Interpret functions that arise in applications in terms of the context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	8-12
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
	F-IF.C – Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F-BF.B – Build new functions from existing functions	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	
		Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.	
Linear, Quadratic, Exponential, and Trigonometric Functions	F-LE.A – Construct and compare linear, quadratic, and exponential model sand solve problems	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	6-10
	F-TF.A – Extend the domain of trigonometric functions using the unit circle	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	
		Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	
	F-TF.B – Model periodic phenomena with trigonometric functions	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	
	F-TF.C – Prove and apply trigonometric identities	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	

Cluster	Standards		# of Items
Geometry: Congruence and Constructions	G-CO.D – Make geometric constructions	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).	5-9
		Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	
	G-C.A – Understand and apply theorems about circles	Prove that all circles are similar.	
		Identify and describe relationships among inscribed angles, radii, and chords.	
		Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	
	G-C.B – Find arc lengths and areas of sectors of circles	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	
Geometry: Properties, Dimension, and Modeling	G-GPE.A – Translate between the geometric description and the equation for a conic section	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	8-12
		Derive the equation of a parabola given a focus and directrix.	
	G-GPE.B – Use coordinates to prove simple geometric theorems algebraically	Use coordinates to prove simple geometric theorems algebraically.	
		Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	
		Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
		Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	
	G-MG.A – Apply geometric concepts in modeling situations	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	
		Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	
		Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	
	G-GMD – Visualize relationships between two-dimensional and three-dimensional objects	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	

Cluster	Standards		# of Items
Interpreting Data, Making Inferences and Justifying Conclusions	S-ID.A – Summarize, represent, and interpret data on a single count or measurement variable	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	9-11
	S-ID.B – Summarize, represent, and interpret data on two categorical and quantitative variables	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. b. Informally assess the fit of a function by plotting and analyzing residuals.	
	S-IC.A – Understand and evaluate random processes underlying statistical experiments	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	
		Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.	
	S-IC.B – Make inferences and justify conclusions from sample surveys, and observational studies	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	
		Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	
		Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	
		Evaluate reports based on data.	

TNReady Algebra I Blueprint

Clusters on Part I	# of items in Part I	% of Part I	Additional Clusters on Part II (All Part I Clusters will also be assessed on Part II)	# of items in Part II	% of Part II	% of test
Structure and operations with expressions and quantities • Properties of rational and irrational numbers • Use units to solve problems • Interpret the structure of expressions • Write expressions in equivalent forms • Perform arithmetic operations on polynomials • Relationships between zeros and polynomial factors	7-9	29-31%	No additional clusters	7-9	19-21%	23-25%
Create equations that describe numbers or relationships	4-6	17-19%	No additional clusters	2-4	7-9%	11-13%
Reason with equations and inequalities • Understand solving equations • Solve equations and inequalities	2-4	12-14%	**Reason with equations and inequalities** • Solve systems of equations • Represent and solve equations and inequalities graphically	8-10	21-23%	17-19%
Interpreting and building functions • Understand the concept of a function and use function notation • Interpret functions that arise in applications in terms of the context • Analyze functions using different representations	6-8	27-29%	**Interpreting and building functions** • Build new functions from existing functions	8-10	22-24%	23-25%
Construct and compare linear, quadratic and exponential functions • Construct and compare linear, quadratic, and exponential models and solve problems	2-4	12-14%	**Construct and compare linear, quadratic and exponential functions** • Interpret expressions for functions in terms of the situation they model	2-4	7-9%	8-10%
No content from these clusters will be assessed on Part I	0	0%	**Interpreting categorical and quantitative data** • Summarize, represent, and interpret data on a single count or measurement variable • Summarize, represent, and interpret data on two categorical and quantitative variables • Interpret linear models	6-8	18-20%	9-11%
Total	21-31	100%	Total	33-45	100%	100%

Cluster	Standards		# of Items
Structure and operations with expressions and quantities	N-RN – Use properties of rational and irrational numbers	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	7-9
	N-Q – Reason quantitatively and use units to solve problems	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	
		Define appropriate quantities for the purpose of descriptive modeling.	
		Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	
	A-SSE.A – Interpret the structure of expressions	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	
		Use the structure of an expression to identify ways to rewrite it.	
	A-SSE.B – Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions.	
	A-APR.A – Perform arithmetic operations on polynomials	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
	A-APR.B – Understand the relationship between zeros and factors of polynomials	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Create equations that describe numbers or relationships	A-CED – Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	4-6
		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
		Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	
		Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	

Reason with equations and inequalities	A-REI.A – Understand solving equations as a process of reasoning and explain the reasoning	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	2-4
	A-REI.B – Solve equations and inequalities in one variable	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
		Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	
Interpreting and building functions	F-IF.A – Understand the concept of a function and use function notation	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	
		Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
		Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	
	F-IF.B – Interpret functions that arise in applications in terms of context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	
		Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
	F-IF.C – Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	6-8
		Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	

	F-BF.A – Build a function that models a relationship between two quantities	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
Construct and compare linear, quadratic and exponential functions	F-LE.A – Construct and compare linear, quadratic, and exponential models and solve problems	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	2-4
		Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
		Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	

Part II – Calculator and Non-Calculator Portions

Cluster	Standards		# of Items
Structure and operations with expressions and quantities	N-RN – Use properties of rational and irrational numbers	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	7-9
	N-Q – Reason quantitatively and use units to solve problems	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	
		Define appropriate quantities for the purpose of descriptive modeling.	
		Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	
	A-SSE.A – Interpret the structure of expressions	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	
		Use the structure of an expression to identify ways to rewrite it.	
	A-SSE.B – Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions.	
	A-APR.A – Perform arithmetic operations on polynomials	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
	A-APR.B – Understand the relationship between zeros and factors of polynomials	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Create equations that describe numbers or relationships	A-CED – Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	2-4
		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
		Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	
		Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	

	A-REI.A – Understand solving equations as a process of reasoning and explain the reasoning	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
Reason with equations and inequalities	A.REI.B – Solve equations and inequalities in one variable	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**8-10**
		Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	
	A-REI.C – Solve systems of equations	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
		Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
	A-REI.D – Represent and solve equations and inequalities graphically	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
		Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	
		Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	
Interpreting and building functions	F-IF.A – Understand the concept of a function and use function notation	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	**8-10**
		Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
		Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	

	F-IF.B – Interpret functions that arise in applications in terms of context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	
		Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
	F-IF.C – Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	
		Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F-BF.A – Build a function that models a relationship between two quantities	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
	F-BF.B – Build new functions from existing functions	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x +k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	
Construct and compare linear, quadratic and exponential functions	F-LE.A – Construct and compare linear, quadratic, and exponential models and solve problems	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	2-4
		Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
		Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
	F-LE.B – Interpret expressions for functions in terms of the situation they model	Interpret the parameters in a linear or exponential function in terms of a context.	

Interpreting categorical and quantitative data	S-ID.A – Summarize, represent, and interpret data on a single count or measurement variable	Represent data with plots on the real number line (dot plots, histograms, and box plots).	6-8
		Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	
		Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	
	S-ID.B – Summarize, represent, and interpret data on two categorical and quantitative variables	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	
		Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	
	S-ID.C – Interpret linear models	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	
		Compute (using technology) and interpret the correlation coefficient of a linear fit.	
		Distinguish between correlation and causation.	

Cluster	Standards		# of Items
Structure and operations with expressions and quantities	N-RN – Use properties of rational and irrational numbers	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	14-18
	N-Q – Reason quantitatively and use units to solve problems	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	
		Define appropriate quantities for the purpose of descriptive modeling.	
		Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	

	A-SSE.A – Interpret the structure of expressions	Interpret expressions that represent a quantity in terms of its context.
		    a. Interpret parts of an expression, such as terms, factors, and coefficients.
		    b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	

		Use the structure of an expression to identify ways to rewrite it.	

	A-SSE.B – Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
		    a. Factor a quadratic expression to reveal the zeros of the function it defines.
		    b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
		    c. Use the properties of exponents to transform expressions for exponential functions.	

	A-APR.A – Perform arithmetic operations on polynomials	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
	A-APR.B – Understand the relationship between zeros and factors of polynomials	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Create equations that describe numbers or relationships	A-CED – Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	6-10
		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
		Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	
		Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	

Reason with equations and inequalities	A-REI.A – Understand solving equations as a process of reasoning and explain the reasoning	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	10-14
	A.REI.B – Solve equations and inequalities in one variable	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	

		Solve quadratic equations in one variable.
		  a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
		  b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	

	A-REI.C – Solve systems of equations	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
		Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
	A-REI.D – Represent and solve equations and inequalities graphically	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
		Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	
		Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	
Interpreting and building functions	F-IF.A – Understand the concept of a function and use function notation	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	14-18
		Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
		Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	

	F-IF.B – Interpret functions that arise in applications in terms of context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	
		Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
	F-IF.C – Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	
		Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F-BF.A – Build a function that models a relationship between two quantities	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
	F-BF.B – Build new functions from existing functions	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x +k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	
Construct and compare linear, quadratic and exponential functions	F-LE.A – Construct and compare linear, quadratic, and exponential models and solve problems	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	4-8
		Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
		Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
	F-LE.B – Interpret expressions for functions in terms of the situation they model	Interpret the parameters in a linear or exponential function in terms of a context.	

Interpreting categorical and quantitative data	S-ID.A – Summarize, represent, and interpret data on a single count or measurement variable	Represent data with plots on the real number line (dot plots, histograms, and box plots).	6-8
		Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	
		Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	
	S-ID.B – Summarize, represent, and interpret data on two categorical and quantitative variables	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	
		Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	
	S-ID.C – Interpret linear models	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	
		Compute (using technology) and interpret the correlation coefficient of a linear fit.	
		Distinguish between correlation and causation.	

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TNReady Geometry Blueprint

Clusters on Part I	# of items	% of Part I	Additional Clusters on Part II (All Part I Clusters will also be assessed on Part II)	# of items	% of Part II	% of test
Congruence • Experiment with transformations in the plane • Understand congruence in terms of rigid motions • Prove geometric theorems • Make geometric constructions	8-10	34-36%	No additional clusters	11-13	29-31%	30-32%
Similarity, Right Triangles, and Trigonometry • Understand similarity in terms of similarity transformations • Prove theorems involving similarity • Define trigonometric ratios and solve problems involving right triangles	9-11	37-39%	No additional clusters	10-12	26-28%	30-32%
No content from this cluster will be assessed on Part I	0	0%	Circles • Understand and apply theorems about circles • Find arc lengths and areas of sectors of circles	5-7	14-16%	8-10%
Geometry: Properties, Dimension, and Modeling • Use coordinates to prove simple geometric theorems algebraically • Explain volume formulas and use them to solve problems • Apply geometric concepts in modeling situations	6-8	27-29%	Geometry: Properties, Dimension, and Modeling • Translate between the geometric description and the equation for a conic section • Visualize relationships between two-dimensional and three-dimensional objects	10-12	27-29%	26-28%
Total	23-29	100%	Total	36-44	100%	100%

Cluster	Standards		# of Items
Congruence	G-CO.A – Experiment with transformations in the plane	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	8-10
		Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
		Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
		Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
		Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
	G-CO.B – Understand congruence in terms of rigid motions	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	
		Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
		Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
	G-CO.C – Prove geometric theorems	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	
		Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	
		Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	
	G-CO.D – Make geometric constructions	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	
		Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	

Cluster	Standards		# of Items
Similarity, Right Triangles, and Trigonometry	G-SRT.A – Understand similarity in terms of similarity transformations	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	9-11
		Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
		Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
	G-SRT.B – Prove theorems involving similarity	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	
		Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
	G-SRT.C – Define trigonometric ratios and solve problems involving right triangles	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
		Explain and use the relationship between the sine and cosine of complementary angles.	
		Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	
Geometry: Properties, Dimension, and Modeling	G-GPE.B – Use coordinates to prove simple geometric theorems algebraically	Use coordinates to prove simple geometric theorems algebraically.	6-8
		Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems(e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	
		Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
		Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	
	G-GMD.A – Explain volume formulas and use them to solve problems	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	
		Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	
	G-MG.A – Apply geometric concepts in	Use geometric shapes, their measures, and their properties to describe objects. (e.g., modeling a tree trunk or a human torso as a cylinder).	
		Apply concepts of density based on area and volume in modeling situations. (e.g., persons per square mile, BTUs per cubic foot).	

Cluster	Standards	193		# of Items
	modeling situations	Apply geometric methods to solve design problems. (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).		

Part II – Calculator and Non-Calculator Portions

Cluster	Standards		# of Items
Congruence	G-CO.A – Experiment with transformations in the plane	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	11-13
		Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
		Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
		Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
		Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
	G-CO.B – Understand congruence in terms of rigid motions	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	
		Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
		Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
	G-CO.C – Prove geometric theorems	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	
		Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	
		Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	
	G-CO.D – Make geometric constructions	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	
		Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	

Cluster	Standards		# of Items
Similarity, Right Triangles, and Trigonometry	G-SRT.A – Understand similarity in terms of similarity transformations	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	10-12
		Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
		Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
	G-SRT.B – Prove theorems involving similarity	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	
		Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
	G-SRT.C – Define trigonometric ratios and solve problems involving right triangles	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
		Explain and use the relationship between the sine and cosine of complementary angles.	
		Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	
Circles	G-C.A – Understand and apply theorems about circles	Prove that all circles are similar.	5-7
		Identify and describe relationships among inscribed angles, radii, and chords.	
		Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	
	G-C.B – Find arc lengths and areas of sectors of circles	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	
Geometry: Properties, Dimension, and Modeling	G-GPE.B – Use coordinates to prove simple geometric theorems algebraically	Use coordinates to prove simple geometric theorems algebraically.	10-12
		Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	
		Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
		Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	
	G-GMD.A – Explain volume formulas and	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	

Cluster	Standards		# of Items
	use them to solve problems	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	
	G-MG.A – Apply geometric concepts in modeling situations	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	
		Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	
		Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	
	G-GPE.A – Translate between the geometric description and the equation for a conic section	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	
	G-GMD.B – Visualize relationships between two-dimensional and three-dimensional objects	Identify the shapes of two-dimensional cross-sections of three dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	

Overall Blueprint (Includes Part I and Part II)

Cluster	Standards		# of Items
Congruence	G-CO.A – Experiment with transformations in the plane	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	19-23
		Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
		Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
		Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
		Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
	G-CO.B – Understand congruence in terms of rigid motions	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	
		Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
		Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
	G-CO.C – Prove geometric theorems	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	
		Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	
		Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	
	G-CO.D – Make geometric constructions	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	
		Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	

Cluster	Standards		# of Items
Similarity, Right Triangles, and Trigonometry	G-SRT.A – Understand similarity in terms of similarity transformations	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	19-23
		Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
		Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
	G-SRT.B – Prove theorems involving similarity	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	
		Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
	G-SRT.C – Define trigonometric ratios and solve problems involving right triangles	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
		Explain and use the relationship between the sine and cosine of complementary angles.	
		Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	
Circles	G-C.A – Understand and apply theorems about circles	Prove that all circles are similar.	5-7
		Identify and describe relationships among inscribed angles, radii, and chords.	
		Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	
	G-C.B – Find arc lengths and areas of sectors of circles	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	
Geometry: Properties, Dimension, and Modeling	G-GPE.B – Use coordinates to prove simple geometric theorems algebraically	Use coordinates to prove simple geometric theorems algebraically.	16-20
		Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	
		Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
		Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	

Cluster	Standards		# of Items
	G-GMD.A – Explain volume formulas and use them to solve problems	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	
		Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	
	G-MG.A – Apply geometric concepts in modeling situations	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	
		Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	
		Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	
	G-GPE.A – Translate between the geometric description and the equation for a conic section	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	
	G-GMD.B – Visualize relationships between two-dimensional and three-dimensional objects	Identify the shapes of two-dimensional cross-sections of three dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	

TNReady Algebra II Blueprint

Clusters on Part I	# of items	% of Part I	Additional Clusters on Part II (All Part I Clusters will also be assessed on Part II)	# of items	% of Part II	% of test
Number Systems: Real and Complex • Extend the properties of exponents to rational exponents. • Perform arithmetic operations with complex numbers. • Use complex numbers in polynomial identities and equations.	1-3	7-9%	No additional clusters	1-3	4-6%	5-7%
Structure and Operations with Expressions and Quantities • Reason quantitatively and use units to solve problems. • Interpret the structure of expressions • Write expressions in equivalent forms to solve problems • Understand the relationship • between zeros and factors of Polynomials • Use polynomial identities to solve problems • Rewrite rational expressions	6-8	24-26%	No additional clusters	5-7	14-16%	18-20%
Creating and Reasoning with Equations and Inequalities • Create equations that describe numbers or relationships • Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Solve systems of equations • Represent and solve equations and inequalities graphically	5-7	22-24%	No additional clusters	4-6	11-13%	15-17%
Interpreting and Building Functions • Understand the concept of a function and use function notation • Interpret functions that arise in applications in terms of the context • Analyze functions using different representations • Build a function that models a relationship between two quantities	7-9	29-31%	No additional clusters	6-8	17-19%	21-23%

• Build new functions from existing functions			201			
Linear, Quadratic, Exponential and Trigonometric Functions and Conic Equations • Construct and compare linear, quadratic, and exponential models and solve problems • Translate between the geometric description and the equation for a conic section	1-3	4-6%	Linear, Quadratic, Exponential and Trigonometric Functions and Conic Equations • Interpret expressions for functions in terms of the situation they model • Extend the domain of trigonometric functions using the unit circle • Model periodic phenomena with trigonometric functions • Prove and apply trigonometric identities	8-10	21-23%	14-16%
Interpreting Data, Making Inferences and Justifying Conclusions • Understand and evaluate random processes underlying statistical experiments • Make inferences and justify • conclusions from sample • surveys, experiments, and observational studies	2-4	9-11%	Interpreting Data, Making Inferences and Justifying Conclusions • Summarize, represent, and interpret data on a single count or measurement variable • Summarize, represent, and interpret data on two categorical and quantitative variables • Understand independence and conditional probability and use them to interpret data • Use the rules of probability to compute probabilities of compound events in a uniform probability model	10-12	27-29%	20-22%
Total	22-34	100%	Total	34-46	100%	100%

Part I – Calculator Allowed

Cluster	Standards		# of Items
Number Systems: Real and Complex	N.RN.A- Extend the properties of exponents to rational exponents	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	1-3
		Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
	N.CN.A- Perform arithmetic operations with complex numbers.	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	
		Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
	N.CN.C- Use complex numbers in polynomial identities and equations.	Solve quadratic equations with real coefficients that have complex solutions.	
Structure and Operations with Expressions and Quantity	N-Q.A- Reason quantitatively and use units to solve problems.	Define appropriate quantities for the purpose of descriptive modeling.	6-8
	A.SSE.A- Interpret the structure of expressions	Use the structure of an expression to identify ways to rewrite it.	
	A.SSE.B- Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions.	
		Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.	
	A-APR.A- Understand the relationship between zeros and factors of polynomials	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	
		Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
	A-APR.B- Use polynomial identities to solve problems	Prove polynomial identities and use them to describe numerical relationships.	

Cluster	Standards		# of Items
	A-APR.C-Rewrite rational expressions	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	
Creating and Reasoning with Equations and Inequalities	A-CED-Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	5-7
	A.REI.A-Understand solving equations as a process of reasoning and explain the reasoning	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
		Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
	A.REI.B-Solve equations and inequalities in one variable	Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	
	A.REI.C-Solve systems of equations	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
		Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	
	A.REI.D-Represent and solve equations and inequalities graphically	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	
Interpreting and Building Functions	F.IF.A-Understand the concept of a function and use function notation	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	7-9
	F.IF.B-Interpret functions that arise in applications in terms of the context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	

Cluster	Standards		# of Items
	F.IF.C- Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
		Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F.BF.A- Build a function that models a relationship between two quantities	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations.	
		Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	
	F.BF.B- Build new functions from existing functions	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	
		Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.	
Linear, Quadratic, Exponential, and Trigonometric Functions and Conic Equations	F.LE- Construct and compare linear, quadratic, and exponential models and solve problems	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	1-3
		For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	
	G-GPE- Translate between the geometric description and the equation for a conic section	Derive the equation of a parabola given a focus and directrix.	
Interpreting Data, Making Inferences and Justifying Conclusions	S.IC.A- Understand and evaluate random processes underlying statistical experiments	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	2-4
		Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.	
	S.IC.B- Make	Recognize the purposes of and differences among sample surveys, experiments,	

Cluster	Standards		# of Items
	inferences and justify conclusions from sample surveys, experiments, and observational studies	and observational studies; explain how randomization relates to each.	
		Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	
		Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	
		Evaluate reports based on data.	

Part II – Calculator and Non-Calculator Portions

Cluster	Standards		# of Items
Number Systems: Real and Complex	N.RN.A- Extend the properties of exponents to rational exponents	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	1-3
		Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
	N.CN.A- Perform arithmetic operations with complex numbers.	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	
		Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
	N.CN.C- Use complex numbers in polynomial identities and equations.	Solve quadratic equations with real coefficients that have complex solutions.	
Structure and Operations with Expressions and Quantity	N-Q- Reason quantitatively and use units to solve problems.	Define appropriate quantities for the purpose of descriptive modeling.	5-7
	A.SSE.A- Interpret the structure of expressions	Use the structure of an expression to identify ways to rewrite it.	
	A.SSE.B- Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions.	
		Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.	
	A-APR.A- Understand the relationship between zeros and factors of Polynomials	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	
		Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
	A-APR.B- Use polynomial identities to solve problems	Prove polynomial identities and use them to describe numerical relationships.	

Cluster	Standards		# of Items
	A-APR.C- Rewrite rational expressions	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	
Creating and Reasoning with Equations and Inequalities	A-CED- Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	4-6
	A.REI.A- Understand solving equations as a process of reasoning and explain the reasoning	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
		Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
	A.REI.B- Solve equations and inequalities in one variable	Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	
	A.REI.C- Solve systems of equations	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
		Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = –3x and the circle $x^2 + y^2 = 3$.*	
	A.REI.D- Represent and solve equations and inequalities graphically	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	
Interpreting and Building Functions	F.IF.A- Understand the concept of a function and use function notation	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	6-8
	F.IF.B- Interpret functions that arise in applications in terms of the context Analyze	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	

Cluster	Standards		# of Items
	functions using different representations		
	F.IF.C- Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
		Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F.BF.A- Build a function that models a relationship between two quantities	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations.	
		Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	
	F.BF.B- Build new functions from existing functions	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	
		Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.	
Linear, Quadratic, Exponential, and Trigonometric Functions and Conic Equations	F.LE- Construct and compare linear, quadratic, and exponential models and solve problems	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	8-10
		For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	
	G-GPE- Translate between the geometric description and the equation for a conic section	Derive the equation of a parabola given a focus and directrix.	

Cluster	Standards		# of Items
	F.LE- Interpret expressions for functions in terms of the situation they model	Interpret the parameters in a linear or exponential function in terms of a context.	
	F-TF.A- Extend the domain of trigonometric functions using the unit circle	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	
		Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	
	F-TF.B- Model periodic phenomena with trigonometric functions	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	
	F-TF.C- Prove and apply trigonometric identities	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	
Interpreting Data, Making Inferences and Justifying Conclusions	S.IC.A- Understand and evaluate random processes underlying statistical experiments	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	10-12
		Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.	
	S.IC.B- Make inferences and justify conclusions from sample surveys, experiments, and observational studies	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	
		Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	
		Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	
		Evaluate reports based on data.	
	S.ID.A- Summarize, represent, and interpret data on a single count or measurement variable	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	

Cluster	Standards		# of Items
	S-ID.B- Summarize, represent, and interpret data on two categorical and quantitative variables	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.	
	S.CP.A- Understand independence and conditional probability and use them to interpret data	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	
		Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	
		Understand the conditional probability of A given B as $P(A$ and $B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	
		Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	
		Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	
	S.CP.B- Use the rules of probability to compute probabilities of compound events in a uniform probability model	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.	
		Apply the Addition Rule, $P(A$ or $B) = P(A) + P(B) - P(A$ and $B)$, and interpret the answer in terms of the model.	

Overall Blueprint (Includes Part I and Part II)

Cluster	Standards		# of Items
Number Systems: Real and Complex	N.RN.A- Extend the properties of exponents to rational exponents	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	2-6
		Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
	N.CN.A- Perform arithmetic operations with complex numbers.	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	
		Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
	N.CN.C- Use complex numbers in polynomial identities and equations.	Solve quadratic equations with real coefficients that have complex solutions.	
Structure and Operations with Expressions and Quantity	N-Q- Reason quantitatively and use units to solve problems.	Define appropriate quantities for the purpose of descriptive modeling.	11-15
	A.SSE.A- Interpret the structure of expressions	Use the structure of an expression to identify ways to rewrite it.	
	A.SSE.B- Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions.	
		Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.	
	A-APR.A- Understand the relationship between zeros and factors of Polynomials	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	
		Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
	A-APR.B- Use polynomial identities to solve problems	Prove polynomial identities and use them to describe numerical relationships.	

Cluster	Standards		# of Items
	A-APR.C-Rewrite rational expressions	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	
Creating and Reasoning with Equations and Inequalities	A-CED-Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems.	9-13
	A.REI.A-Understand solving equations as a process of reasoning and explain the reasoning	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
		Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
	A.REI.B-Solve equations and inequalities in one variable	Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	
	A.REI.C-Solve systems of equations	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
		Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	
	A.REI.D-Represent and solve equations and inequalities graphically	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	
Interpreting and Building Functions	F.IF.A-Understand the concept of a function and use function notation	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	13-17
	F.IF.B-Interpret functions that arise in applications in terms of the	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	
		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	

Cluster	Standards		# of Items
	context Analyze functions using different representations		
	F.IF.C- Analyze functions using different representations	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
		Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions.	
		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
	F.BF.A- Build a function that models a relationship between two quantities	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations.	
		Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	
	F.BF.B- Build new functions from existing functions	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	
		Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.	
Linear, Quadratic, Exponential, and Trigonometric Functions and Conic Equations	F.LE- Construct and compare linear, quadratic, and exponential models and solve problems	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	9-13
		For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	
	G-GPE- Translate between the geometric description and the equation for a conic section	Derive the equation of a parabola given a focus and directrix.	

Cluster	Standards		# of Items
	F.LE- Interpret expressions for functions in terms of the situation they model	Interpret the parameters in a linear or exponential function in terms of a context.	
	F-TF.A- Extend the domain of trigonometric functions using the unit circle	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	
		Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	
	F-TF.B- Model periodic phenomena with trigonometric functions	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	
	F-TF.C- Prove and apply trigonometric identities	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	
Interpreting Data, Making Inferences and Justifying Conclusions	S.IC.A- Understand and evaluate random processes underlying statistical experiments	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	12-16
		Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.	
	S.IC.B- Make inferences and justify conclusions from sample surveys, experiments, and observational studies	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	
		Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	
		Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	
		Evaluate reports based on data.	
	S.ID.A- Summarize, represent, and interpret data on a single count or measurement variable	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	

Cluster	Standards	215	# of Items
	S-ID.B-Summarize, represent, and interpret data on two categorical and quantitative variables	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.	
	S.CP.A-Understand independence and conditional probability and use them to interpret data	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	
		Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	
		Understand the conditional probability of *A* given *B* as $P(A \text{ and } B)/P(B)$, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	
		Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	
		Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	
	S.CP.B-Use the rules of probability to compute probabilities of compound events in a uniform probability model	Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.	
		Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.	

Appendix H
Fluency Document

Tennessee State Standards, Fluency in Mathematics

Fluency is not meant to come at the expense of conceptual understanding. Rather, it should be an outcome resulting from a progression of learning and thoughtful practice. It is important to provide the conceptual building blocks that develop understanding along with skill toward developing fluency; the roots of this conceptual understanding often occur one or more grades earlier in the standards than the grade when fluency is expected.

"Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate *flexibility* in the computational methods they choose, *understand* and can explain these methods, and produce accurate answers *efficiently*. The computational methods that a student uses should be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication and division, and number relationships."

NCTM, Principles and Standards for School Mathematics, p. 152 (2000)

Fluency Expectations, K-8

Grade	Standard	Expected Fluency
K	K.OA.A.5	Add/Subtract within 5
1	1.OA.C.6	Add/Subtract within 10
2	2.OA.B.2 2.NBT.B.5	Add/Subtract within 20 (Know single digit sums from memory) Add/Subtract within 100
3	3.OA.C.7* 3.NBT.A.2*	Multiply/Divide within 100 (Know single digit products from memory) Add/Subtract within 1000
4	4.NBT.B.4*	Add/Subtract within 1,000,000
5	5.NBT.B.5*	Multi-digit multiplication
6	6.NS.B.2* 6.NS.B.3*	Multi-digit division Multi-digit decimal operations
7	7.NS.A.1,2 7.EE.B.3 7.EE.B.4	Fluency with rational number arithmetic Solve multistep problems with positive and negative rational numbers in any form Solve one-variable equations of the form $px + q = r$ and $p(x + q) = r$ fluently
8	8.EE.C.7 8.G.C.9	Solve one-variable linear equations, including cases with infinitely many solutions or no solutions Solve problems involving volumes of cones, cylinders, and spheres together with previous geometry work, proportional reasoning and multi-step problem solving in grade 7

*These fluency standards will be assessed on TNReady. Students will not have access to a calculator for fluency items on TNReady.

TNCore

Tennessee State Standards, Fluency in Mathematics

The high school standards do not set explicit expectations for fluency, but fluency is important in high school mathematics. Fluency in algebra can help students get past the need to manage computational and algebraic manipulation details so that they can observe structure and patterns in problems. Such fluency can also allow for smooth progress toward readiness for further study/careers in science, technology, engineering, and mathematics (STEM) fields. These fluencies are highlighted to stress the need to provide sufficient supports and opportunities for practice to help students gain fluency. Fluency is not meant to come at the expense of conceptual understanding. Rather, it should be an outcome resulting from a progression of learning and thoughtful practice. It is important to provide the conceptual building blocks that develop understanding along with skill toward developing fluency

Fluency Recommendations, High School
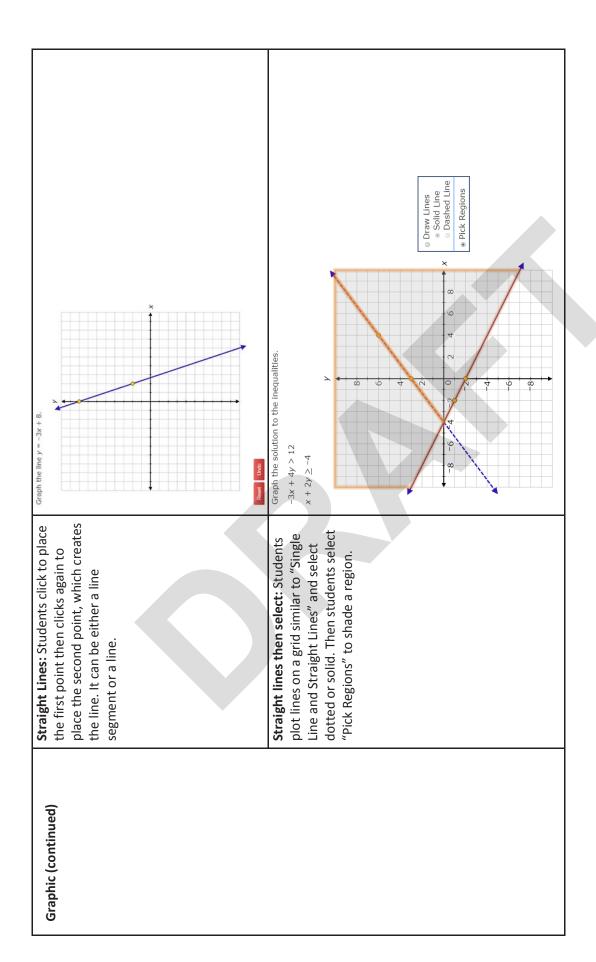
Course	Standard	Recommended Fluency
Algebra I	A/G A-APR.A.1 A-SSE.A.1b	Solving characteristic problems involving the analytic geometry of lines Fluency in adding, subtracting, and multiplying polynomials Fluency in transforming expressions and seeing parts of an expression as a single object
Geometry	G-SRT.B.5 G-GPE.B.4, 5, 7 C-CO.D.12	Fluency with the triangle congruence and similarity criteria Fluency with the use of coordinates Fluency with the use of construction tools
Algebra II	A-APR.D.6 A-SSE.A.2 F.IF.A.3	Divide polynomials with remainder by inspection in simple cases See structure in expressions and use this structure to rewrite expressions Fluency in translating between recursive definitions and closed forms

Appendix I
Item Types

Math Item Type

Item Type	Examples	Illustration
Equation: Students generate response. On both part I and part II. Typically worth one point (one right answer). 10-15% of questions.	**Equation Editor:** Students type in numeric answers from a palette of options.	**5** The manager of a youth soccer team bought 50 packages of socks for $10 each. He estimated the total cost to be $5,000. Create an equation that shows how many times more the manager's estimate, e, was than the actual cost, a. $$a \times 10 = e$$
Graphic: Students depict graphically. On both part I and part II but more often on part I. Usually one point. 10-25% of questions, more in high school.	**Circles:** Students graph a circle by plotting the center point first and then the radius.	Graph the equation $(x - 1)^2 + (y - 2)^2 = 3^2$.

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Graphic (continued)	
Line Graph: Students click to add a point. Adding another point will automatically connect the points to form a line graph.	The table below shows the speed of a bicyclist. Using the data in the table, create a line on the graph showing the bicyclist's speed. **Bicyclist Speed** | Time (s)	Distance (m)
0	0
10	5
20	12
30	12
40	18
50	25
60	36 | *Bicyclist Speed graph — Distance (m) vs. Time (s)* Reset Clear Undo
Placing Points: Students click to add a point.	The data table shows the locations of different buildings in a town. **Town Buildings** | Buildings	Location
Grocery Store (G)	(7, 5)
Library (L)	(7, 1.5)
Park (P)	(3, −1)
School (S)	(−3, 5) | The grid shown represents a town. Plot each point on the coordinate grid. Label each point with the name of the building.
Single Parabola: Students select open-up, open-down, open-left, open-right to place a parabola on the grid. The points or the entire line can then be moved.	Graph the equation $y = -x^2 + 3$ on the coordinate grid. Open-up Open-down Open-left Open-right Reset Undo

Graphic (continued)

Straight Lines: Students click to place the first point then clicks again to place the second point, which creates the line. It can be either a line segment or a line.

Graph the line $y = -3x + 8$.

Reset   Undo

Straight lines then select: Students plot lines on a grid similar to "Single Line and Straight Lines" and select dotted or solid. Then students select "Pick Regions" to shade a region.

Graph the solution to the inequalities.

$-3x + 4y > 12$

$x + 2y \geq -4$

Draw Lines
- Solid Line
- Dashed Line

- Pick Regions

	Vertex based Quadrilaterals: Students plot points to form different polygons. The option can be set to close the shape after four points are plotted. Create a shape on the grid with the following: • 4 sides of equal length • 2 pairs of parallel sides • 4 right angles
Multiple Choice: Students select one answer, only one answer is correct. On both part I and part II. Worth one point. 60-75% of questions.	**Dropdowns:** Students choose from a dropdown list. Use this number to create a true sentence. 684.425 The value of the 4 in the tenths place is [one-tenth of ▼] the value of the 4 in the ones place. **Multiple Choice:** Student selects one correct answer from many options. **1** What is 78 rounded to the nearest ten? Ⓐ 70 Ⓑ 75 ● 80 Ⓓ 100
Multiple select: Students select multiple options, multiple correct. On both part I and part II but more on part II. Can be worth one to two points, depending on the	**Check Box:** Students select multiple correct answer choices. **3** Consider the family of quadrilaterals that includes parallelograms, rectangles, squares, and rhombuses. Select all the statements about these quadrilaterals that are true. ☑ Squares are always rectangles. ☐ Rectangles are always squares. ☐ Rhombuses are always squares. ☑ Squares are always rhombuses. ☑ Rhombuses are always parallelograms. ☑ Rhombuses are sometimes rectangles.

	Matching Table: Students select multiple correct answer choices.	Select one phrase that describes the value of each expression.

wording of the question.

Up to 10% of questions.

Matching Table: Students select multiple correct answer choices.

Select one phrase that describes the value of each expression.

	Greater than 3	Equal to 3	Less than 3
$3 \times \frac{1}{2}$	☐	☐	☑
$3 \times 1\frac{1}{4}$	☑	☐	☐
$3 \times \frac{6}{6}$	☐	☑	☐
$3 \times \frac{3}{2}$	☑	☐	☐

Select Objects: Students click on objects to select. A select border color appears around the objects when selected.

Click on the shapes that are quadrilaterals.

Reset

Performance Tasks: Solve multistep problems and demonstrate how solution is achieved.

1 task on part I only in grades 3-8 only.

Worth 10-15 points with partial credit.

These will not mimic the previous CRA tasks but will require multistep problem solving and will require students to explain the problem solving approach.

Partial credit will be available.

We are waiting on copyright clearance to release an example performance task. A depiction of a similar type of problem will be shared in the power point.

TEI: Students perform an interaction to respond to the question. Usually worth one point. Occasionally involves two parts. With two parts, the scoring can either be worth one point or two points (with partial credit) depending on the wording of the question. On both Part I and Part II. 5-15%	**Angles:** Students plot 3 points to form an angle. The first point plotted is the vertex. Click in the workspace to draw an angle that measures 48°. Use the protractor tool to measure the angle.
	Bar Graph: Students drag bars up or down to place. The table shows the favorite foods of students in a class. **Favorite Food** | Food	Number of Students
hamburger	5
pizza	10
taco	4
hot dog	6 | Complete the bar graph to show the same information.

TEI (continued)	
Classification: Students drag and drop objects to different regions.	**2** The numbers 8 and 6 are added, and the sum is then multiplied by 3. A. Drag numbers to the boxes and symbols to the circles to represent the expression described. B. Drag numbers to the boxes and symbols to the circles to create an equivalent expression to the one you created in part A. 3　6　8　×　+ **A.** $(8 \ + \ 6 \ ) \ \times \ 3$ **B.** $(8 \ \times \ 3 \ ) \ + \ (6 \ \times \ 3)$
Partition Number Lines then Place Points: Students click a button to partition a line into equal parts. They can then select "Place Points" to be able to click on the line to place a point. With Student Label, the student will add partitions to the line instead of using a button to partition equally.	Divide the number line so that a point can be plotted on a tick mark at 22.3. Place a point at the location of 22.3 22　　　　　　　23 Reset　Clear　Undo ○ Partition Line 　Add a Partition Line 　Remove a Partition Line ● Place Points
Partition Object then Select: Students click a button to partition an object into equal parts. They can then select "Shade Regions" to shade sections. In Select Defined partitions, a student will not be partitioning.	Shade one-fourth of the circle. Add Line Remove Line Reset

TEI (continued)		
	Pictograph: Students click a plus or minus to add shapes to the pictograph.	This table shows daily movie sales at the Video Store. **Video Store Movie Sales** | Type of Movie	Number of Movies Sold Per Day
---	---
Action	35
Cartoon	10
Comedy	40
Drama	65 | Complete the pictograph to show the same information. **Video Store Movie Sales** | Type of Movie	Number of Movies Sold Per Day
---	---
Action	
Cartoon	
Comedy	
Drama		 **Key** ◯ = 10 Movies Reset
	Rays: Students plot 2 points to form a ray. The first point is the end point. The second point has the arrow at the end.	
	Select Points And Ranges on Number Lines: Students click on a point on the line to place an open/closed marker. After markers are placed, students can click in between points to select a region. Regions extend between two points or one point and the end of the number line.	Graph the solution to the inequality on the number line. $x > 2\frac{1}{2}$ Reset Clear Undo ● Closed ○ Open
	Shade Regions: Students click to shade regions of a rectangle. Selected regions are either outlined or filled with a solid color, symbol, or custom graphic.	The figure shown is made up of same-size squares. Click the image to shade 0.35 of this figure. Each ☐ = 0.01 Reset Undo

Appendix J
Calculator Policies

TNReady Calculator Policy for Mathematics

The TNReady Calculator Policy is based on two central beliefs:

1) Calculators are important tools and, in order to be ready for career and college, students need to understand how to use calculators effectively, and
2) In order to demonstrate mastery of the mathematics standards, students must demonstrate many skills without reliance on calculators.

Therefore, at all grade levels and in all courses, TNReady will include both calculator permitted sections and calculator prohibited sections.

- Part I will allow calculator use at all grade levels.
- Part II will include a calculator permitted section and a calculator prohibited section at all grade levels.

The following considerations will shape how items are assigned to each section:
- Questions based on standards that require students to perform calculations in order to arrive at an answer will appear on the non-calculator-permitted section of the assessment. For example, 5.NF.A.1 expects students to add/subtract fractions with unlike denominators.
- Other questions may be based on standards where a calculation is a means to demonstrating other understanding. In this case, a student's error could be based on a misconception or a miscalculation, which would color the evidence of what is intended by the assessment. For example, 6.G.A.1 expects students to find area of composite figures and the calculations performed should not be a barrier for students demonstrating understanding of how to determine the area.
- Questions based on standards like 3.G.A.1 which ask students to recognize examples of quadrilaterals may appear on either the calculator or non-calculator section.

Calculator Specifics

- It is the responsibility of the Test Administrator to ensure the regulations outlined in this policy pertaining to calculator use are followed.
- All memory and user-entered programs and documents must be cleared or removed before and after the test.
- A student may use any permitted calculator at any grade level.
- For calculator-permitted sections of TNReady, students may use the online calculator or a handheld calculator provided by the school/district or one owned personally. Students may use either or both during the test.

Draft, 4.8.2015

- Students should have access to no more than one handheld calculator device for calculator-permitted sections of TNReady.
- Students will have access to practice with the same functionalities that will be available on the operational assessment on the item sampler and the practice tests.

Calculator Types

Below are examples of calculator functionalities and calculators that are permitted on TNReady. (Note: this is not an exhaustive list and students should be familiar with particular functions at the appropriate grade level.)

Examples of Permitted Functionalities:

- Square root ($\sqrt{\ }$)/Square key (x^2 and/or x^y)
- Pi(π)
- Graphing capability
- Data entry
- Matrices
- Regression
- Trigonometric functions (sine, cosine, tangent)
- Logarithm (log and/or ln) and exponential functions (a^x and/or e^x)

Examples of permitted calculators:

- TI-30
- Casio FX260
- Sharp EL344RB
- TI-84 plus family
- TI-NSpire (non-CAS) and TI-NSpire-CX (non-CAS)

Below are calculator functionalities and examples of calculators that are not allowed on TNReady. (Students may use any four- function, scientific, or graphing calculator, which does not include any of the prohibited functionalities.)

Calculator functionalities that are prohibited:

- Any calculator with CAS (computer algebra system) capabilities (including any programs or applications)
- Wireless communication capability
- QWERTY keyboard
- Cell phones, tablets, iPods, etc.

Examples of prohibited calculators:

- TI-89
- TI-NSpire (CAS version)
- HP-40G
- Casio CFX-9970

Appendix K
Practice Tools

TNReady Practice Tools

The TDOE will make two optional tools available to educators and districts.

	TNReady *Item Sampler*		TNReady *Practice Test*
	Phase 1	**Phase 2**	
Purpose	• In Phase 1, to give educators access to questions that are reflective of the rigor and the format of questions that will be on TNReady. • In Phase 2, to give educators access to additional items and provide students a chance to practice with the same tools they will have on TNReady in an instructional setting.		• To simulate a short-form of each part of the TNReady test (Part I and Part II). • To allow students to experience a practice test with the same features as the operational assessment. • To allow teachers and systems to practice set up and administration.
Limitations	• The TNReady Item Sampler will not serve as a full set of interim or formative assessments. • The items will not be secure. (All teachers will have access at the same time.) • The items will be comparable to the items on TNReady but the test forms will not be comparable, as they are teacher-created. • The results will not necessarily be comparable to results in other classrooms because the user experience cannot be controlled. • The ability to add customized items specific to teacher, school, or district will not be available at this time.		• Results on the TNReady Practice Test will not necessarily be predictive of student performance on TNReady. • The Practice test will not reflect a full form, but it will include all major item types for each part.
Timeline	• Launch May 2015 • Continuously available • 8-12 items per grade per subject • Full range of item types • Access for teachers only	• Launch September 2015 • Continuously available • 25-40 additional items per grade per subject • Full range of standards • Access for teachers and students	• <u>Window 1</u>: September 28 – October 30, 2015 (All grades Part I & II) • <u>Window 2</u>: January 4 – February 6, 2016 (All grades Part I & II) • <u>Window 3</u>: March 7 – April 8, 2016 (All grades Part II)
User Set-Up	• All teachers will be set up to get access on May 2015. • More info about set up will be shared with testing coordinators by April 15, 2015.	• Students will be set up based on August 14 EIS pull, provided scheduling data is available. Regular EIS updates thereafter. • More info will be shared with testing coordinators by August 1, 2015.	• Same upload process as MIST practice test has been in the past.
Training	• Training on MICA will be incorporated in summer training for teachers and principals. • Web-based video training will be available. • Additional support from CORE analysts.		• Web-based video training will be available.

Reporting	• Teacher Reports
	  o Student, Assessment, Class
	  o Content strand summary
		• Teacher Reports (same as Phase I)
		• Administrator Reports
		  o Utilization by teacher and content strand
		  o "Impersonation" at teacher level
			• Student Report and Roster Report for schools and districts including:
			  o Overall Score and By Standard
			  o Raw Score (number correct)
			  o % correct

Platform	• MICA (Measurement Inc. Classroom Assessments) which is built to reflect MIST.		• MIST (the same platform as the operational assessment).
Accessibility Features	• Supports common web-browser text reader tools. Not integrated in MICA.		• Text reader availability by Windows 2 and 3.
Scoring	• All *machine scorable* items will be automatically scored within platform. • Teachers will be able to go into the MICA system to score student answers to open-response items using the same tools and scoring guides that will be used to score TNReady.		
Cost	• Provided to all Tennessee districts at no additional charge.		

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TN Task Resources

Appendix L
Task Packet

TCAP/CRA

2014

A1
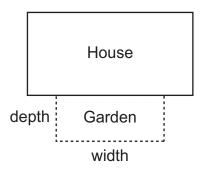
Phase II

Vegetable Garden Task

Anchor Set

Constructed Response Assessment

Vegetable Garden Task

Clariece is fencing in a rectangular garden in her back yard for growing vegetables. She has 18 feet of fence to use. She is using her house to bound one side of the garden, so she only needs to use the fencing for 3 sides, as shown in the diagram below.

The equation $18 = 2d + w$ can be used to represent the relationship between the depth and width of the garden and the total amount of fencing.

a.   Rearrange the equation so that depth is expressed as a function of width. Show your work.

b.   Solve the equation to find the depth of the vegetable garden if the width is 6.5 feet.

Vegetable Garden Task

Clariece wants the fence to enclose the largest possible area for her garden. She creates the graph below to represent the relationship between the width of the garden in feet and the area of the garden in square feet.

c.   What are the depth and width of the garden with the largest possible area? Explain your reasoning.

The CCSS for Mathematical Content Addressed In This Task

Creating Equations	A-CED
Create equations that describe numbers or relationships.	
A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.
Reasoning with Equations and Inequalities	**A-REI**
Solve equations and inequalities in one variable.	
A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
Interpreting Functions	**F-IF**
Interpret functions that arise in applications in terms of the context.	
F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

The CCSS for Mathematical Practice*

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

* Gray type indicates Mathematical Practices not addressed in this assessment.

Appendix M
Task Arc

Mathematics Task Arcs

Overview of Mathematics Task Arcs:

A task arc is a set of related lessons which consists of eight tasks and their associated lesson guides. The lessons are focused on a small number of standards within a domain of the Tennessee State Standards for Mathematics. In some cases, a small number of related standards from more than one domain may be addressed.

A unique aspect of the task arc is the identification of essential understandings of mathematics. An essential understanding is the underlying mathematical truth in the lesson. The essential understandings are critical later in the lesson guides, because of the solution paths and the discussion questions outlined in the share, discuss, and analyze phase of the lesson are driven by the essential understandings.

The Lesson Progression Chart found in each task arc outlines the growing focus of content to be studied and the strategies and representations students may use. The lessons are sequenced in deliberate and intentional ways and are designed to be implemented in their entirety. It is possible for students to develop a deep understanding of concepts because a small number of standards are targeted. Lesson concepts remain the same as the lessons progress; however the context or representations change.
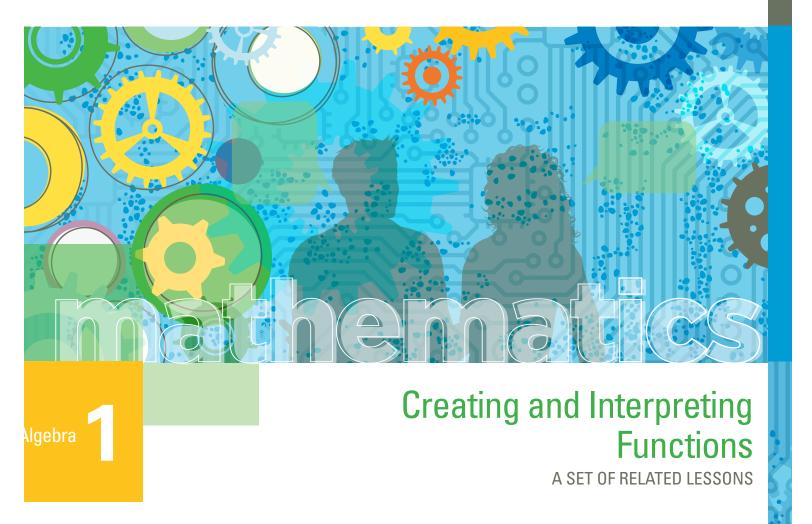
Bias and sensitivity:

These tasks are peer-reviewed and have been vetted for content by district-level experts. However, it is the responsibility of local school districts to review these tasks for bias before use in local schools.

 Copyright:

These task arcs have been purchased and licensed indefinitely for the exclusive use of Tennessee educators.

institute for learning

mathematics

Algebra **1**

Creating and Interpreting Functions

A SET OF RELATED LESSONS

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Table of Contents

mathematics

Algebra **1**

Introduction
Creating and Interpreting Functions
A SET OF RELATED LESSONS

Overview

In this set of related lessons, students create and interpret functions, rate of change, and average rate of change of the function. In the first three tasks, students create linear functions, focusing on rate of change, as well as domain and range. The tasks start with a context before moving to graphical representations. Students solidify their understanding of these concepts in Task 4 before moving to more complex problem situations in Tasks 5 - 8. In these tasks, students interpret average rate of change of qualitative graphs. Task 5 begins with a graphical representation that students interpret. In Task 6, two situations are given that students must graph and interpret. In Task 7, students compare rate of change and average rate of change of two qualitative graphs on the same coordinate plane. Students solidify this understanding in Task 8. All eight tasks are grounded in the study of rate of change of functions in various contexts and representations.

The tasks are aligned to the A.CED.A.2, F.IF.B.4, F.IF.B.5, and F.IF.B.6 Content Standards of the CCSSM.

The prerequisite knowledge necessary to enter these lessons is an understanding of how to evaluate and graph a function given an algebraic representation.

Through engaging in the lessons in this set of related tasks, students will:

- create and interpret functions;
- calculate and interpret rate of change and average rate of change; and
- interpret the properties of functions graphed on the coordinate plane.

By the end of these lessons, students will be able to answer the following overarching questions:
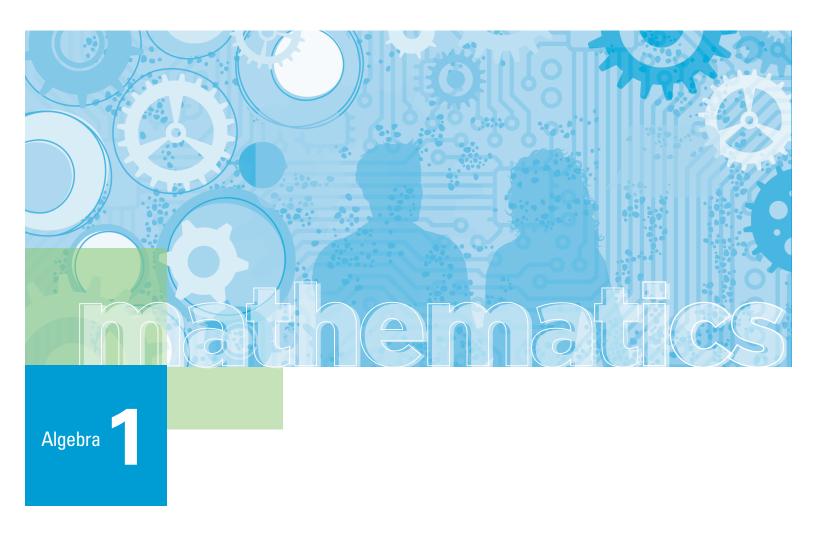
- How can a linear model describe how one variable changes with respect to the other?
- How is rate of change calculated and interpreted graphically?
- What is average rate of change and how is it the same/different from simple linear rate of change?

The questions provided in the guide will make it possible for students to work in ways consistent with the Standards for Mathematical Practice. It is not the Institute for Learning's expectation that students will name the Standards for Mathematical Practice. Instead, the teacher can mark agreement and disagreement of mathematical reasoning or identify characteristics of a good explanation (MP3). The teacher can note and mark times when students independently provide an equation and then re-contextualize the equation in the context of the situational problem (MP2). The teacher might also ask students to reflect on the benefit of using repeated reasoning, as this may help them understand the value of this mathematical practice in helping them see patterns and relationships (MP8). In study groups, topics such as these should be discussed regularly because the lesson guides have been designed with these ideas in mind. You and your colleagues may consider labeling the questions in the guide with the Standards for Mathematical Practice.

Identified CCSSM and Essential Understandings

CCSS for Mathematical Content: Creating Equations and Interpreting Functions		Essential Understandings
Create equations that describe numbers or relationships.		
A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	The solution to an equation can be represented algebraically, graphically, in a table, and in a context. Linear equations can be used to model real-world situations. Equations describing real-world contexts require special attention to units when reasoning algebraically and performing calculations. The graph of a linear relationship is a line that models the relationship between the variables. The coordinates of the points on the line form the solution set for the associated linear equation.
Interpret functions that arise in applications in terms of the context.		
F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values.

CCSS for Mathematical Content: Creating Equations and Interpreting Functions		Essential Understandings
F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	In certain contexts, the domain and range of the relationship that models the context are constrained by the context because certain values may not make sense in the situation.
F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	The average rate of change is the ratio of the change in the dependent variable over a specified interval in the domain to the change in the independent variable over the same interval.

The CCSS for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*Common Core State Standards, 2010, NGA Center/CCSSO

Tasks' CCSSM Alignment

Task	A.CED.A.2	F.IF.B.4	F.IF.B.5	F.IF.B.6
Task 1 **Joe's on the Beach** **Ice Cream** Developing Understanding	✓		✓	
Task 2 **Jose's Surfboard** Developing Understanding	✓	✓	✓	
Task 3 **Ocoee Sand Dunes** Developing Understanding	✓	✓	✓	
Task 4 **More Sand Dunes** Solidifying Understanding	✓	✓		
Task 5 **Swimming Pool Depth** Developing Understanding		✓		✓
Task 6 **Walking** Developing Understanding	✓	✓		✓
Task 7 **Bike and Truck** Developing Understanding		✓		✓
Task 8 **Sandpiper vs. the** **Sand Crab** Solidifying Understanding				✓

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Task	MP 1	MP 2	MP 3	MP 4	MP 5	MP 6	MP 7	MP 8
Task 1 Joe's on the Beach Ice Cream Developing Understanding	✓	✓		✓		✓	✓	
Task 2 Jose's Surfboard Developing Understanding	✓			✓		✓	✓	
Task 3 Ocoee Sand Dunes Developing Understanding	✓		✓	✓		✓	✓	
Task 4 More Sand Dunes Solidifying Understanding	✓	✓	✓	✓		✓	✓	✓
Task 5 Swimming Pool Depth Developing Understanding	✓	✓	✓	✓		✓	✓	
Task 6 Walking Developing Understanding	✓	✓		✓		✓		
Task 7 Bike and Truck Developing Understanding	✓	✓	✓	✓		✓		
Task 8 Sandpiper vs. the Sand Crab Solidifying Understanding	✓		✓	✓		✓		✓

Lesson Progression Chart

Overarching Questions
- How can a linear model describe how one variable changes with respect to the other?
- How is rate of change calculated and interpreted graphically?
- What is the average rate of change and how is it the same/different from simple linear rate of change?

	Task 1 Joe's on the Beach Ice Cream *Developing Understanding*	Task 2 Jose's Surfboard *Developing Understanding*	Task 3 Ocoee Sand Dunes *Developing Understanding*	Task 4 More Sand Dunes *Solidifying Understanding*
Content	Create linear functions in two variables to model a graph.	Create linear functions in two variables to model a graph. Analyze rate of change.	Determine whether a given function correctly models a situation. Analyze domain and range of a function.	Solidify understanding of rate of change from a graph.
Strategy	Substitute values into the equation; determine how change in one variable affects the other.	Analyze change using the formula or other method of determining change in y compared to change in x.	Substitute values; use the slope formula.	Determine the change in y-values compared to the change in x-values between two points on the coordinate plane.
Representations	Starts with a context; student writes an equation and represents the situation graphically.	Starts with a graph; student interprets the graph and writes an equation.	Starts with points on the coordinate plane; student analyzes an equation to determine whether it correctly models the graph.	Starts with graphical representation; student represents the rate of change symbolically.

	Task 5 **Swimming Pool Depth** *Developing Understanding*	Task 6 **Walking** *Developing Understanding*	Task 7 **Bike and Truck** *Developing Understanding*	Task 8 **Sandpiper vs. the Sand Crab** *Solidifying Understanding*
Content	Create and solve two-variable linear equations; analyze a situation over specific intervals.	Sketch functions given the rates of change over specific intervals.	Describe key characteristics of a graph.	Solidify an understanding of average rate of change.
Strategy	Analyze the rate of change graphically.	Compare the rates of change for two different situations graphically, using a formula.	Analyze steepness over specific intervals using a formula, slope triangle, or comparing vertical and horizontal change.	Calculate average rate of change using specific intervals.
Representations	Starts with a table.	Starts with a written description; student creates the graphs.	Starts with a graph that student interprets.	Starts with two graphs that student interprets. Student then sketches graphs given an average rate of change.

mathematics

Algebra **1**

Tasks and Lesson Guides
Creating and Interpreting Functions
A SET OF RELATED LESSONS
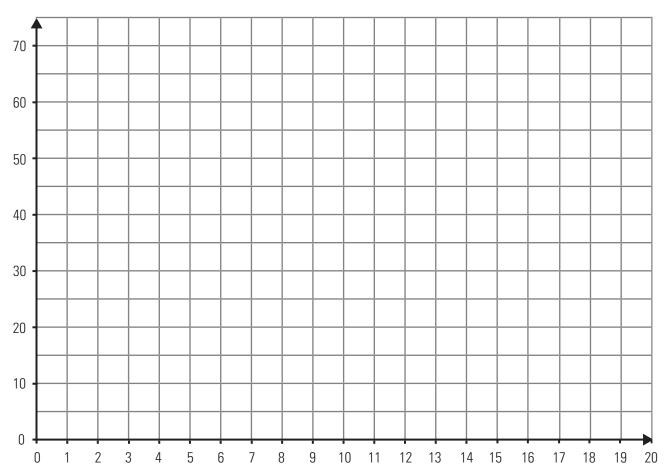
Name_____

Joe's on the Beach Ice Cream

At Joe's on the Beach, single-scoop ice cream cones sell for $2.99 and ice cream cakes sell for $24.99. Rosa buys an ice cream cake for her party. She also decides to buy a single-scoop cone for each of her friends.

1. Write a function that can be used to determine the cost *(y)* of a cake and any number of cones *(x)* that Rosa buys. Explain the meaning of the terms in your function.

2. Sketch a graph that models the problem situation. Explain how you know your graph models the problem situation.

3. How does the total cost increase with the number of cones bought? How does this appear in the function and the graph?

Joe's on the Beach Ice Cream

Rationale for Lesson: Models a situation using a function and a graph. Describes rate of change in a context. Develop an understanding of how context can affect the domain of a function.

Task 1: Joe's on the Beach Ice Cream
At Joe's on the Beach, single-scoop ice cream cones sell for $2.99 and ice cream cakes sell for $24.99. Rosa buys an ice cream cake for her party. She also decides to buy a single-scoop cone for each of her friends.
1. Write a function that can be used to determine the cost (y) of a cake and any number of cones (x) that Rosa buys. Explain the meaning of the terms in your function.
2. Sketch a graph that models the problem situation. Explain how you know your graph models the problem situation.

See student paper for complete task.

Tennessee State Standards	A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP6 Attend to precision. MP7 Look for and make use of structure.	
Essential Understandings	• Linear equations can be used to model real-world situations. Equations describing real-world contexts require special attention to units when reasoning algebraically and performing calculations. • The graph of a linear relationship is a line that models the relationship between the variables in the context. The coordinates of the points on the line form the solution set for the associated linear equation. • In certain contexts, the domain and range of the relationship that models the context are constrained by the context because certain values may not make sense in the situation.	
Materials Needed	• Task. • Extra graph paper. • Straight edge. • Calculator (optional).	

▶ **SET-UP PHASE**

Please read the task out loud. Who can summarize this problem situation without discussing
answers? You will have 5 minutes of private think time before working with your groups.

▶ **EXPLORE PHASE**

EXPLORE PHASE		
Possible Student Pathways	**Assessing Questions**	**Advancing Questions**
Group can't get started.	What do you know about the problem?	How much will she spend if she buys 1 cone (cake included)? 2 cones? What pattern do you notice?
Students set up a table. 	How did you determine the values in your table?	If Rosa buys 50 cones, how will you calculate the cost? X cones?
Sketches graph but is unable to write an equation.	What does the y-intercept represent? What is the rate of change?	What about this problem situation tells us that the graph will be a straight line? How can you use that information to write an equation?
Has a graph and an equation but has difficulty explaining the terms of the equation.	What is the connection between the constant term in your equation and your graph?	How is the cost of a single cone shown in the graph?

Table within "Students set up a table." cell:

# of cones	Cost
0 (may not appear)	24.99
3	33.96
8	48.91

▶ **SHARE, DISCUSS, AND ANALYZE PHASE**

EU: Linear equations can be used to model real-world situations. Equations describing real-world contexts require special attention to units when reasoning algebraically and performing calculations.
- Show us the table that you created and tell us how the table helped you think about the problem. Who can show us the cost per cone in this table? What patterns do we see? Tell us how you used the patterns to generate an equation.
- Say more about how the equation you wrote models the problem situation. What does each term represent? *(The 24.99 represents the cake. The 2.99 represents the cost of each cone. The x is the number of cones and the y is the total cost.)*
- How do you know this equation models the information in the table?
- I'm hearing you say that the constant value is the cake because this only occurs once while the x is the number of cones and this changes by 2.99, depending on how many cones are bought. **(Revoicing)**
- Tell us more about the term "2.99c." Why did you express it this way? Could you have written "299c" in your equation instead of "2.99c"? Why or why not?
- How can this group's equation be used to predict the cost of 4 cones? How does this appear in the table?

EU: The graph of a linear relationship is a line that models the relationship between the variables in the context. The coordinates of the points on the line form the solution set for the associated linear equation.
- How does your group's graph represent the problem situation?
- Who can show us on the graph where the cost per cone is? What about the cost of the ice cream cake? *(The y-intercept shows the cost of the cake and you can see the line increase by close to 3 for every 1 unit over because this is the cost of each cone.)*
- How does this relate to the table and the equation? *(In the table you can see the point [0, 24.99] and a change of 2.99 for every x increase of 1.)*
- How is the pattern that we observed in the table showing up in the graph? How does that pattern relate to the problem context?
- Let's look back at the equation again. How does the structure of the equation help us interpret the graph? *(The equation is linear, in the form y = mx + b, so this lets you know that the graph will be a line.)*
- Do the equation and the graph model the same relationship? Why or why not?
- Let's consider this point here. How do these x and y coordinates relate to the equation? Can somebody restate what this group just said about the coordinates making the equation true?
- Will that be true of every point on this line? Why or why not?
- So we can agree that the function is linear because it is a line and the equation is in the form y = mx + b. We can see that the m represents the rate of change and the b value is the constant term which can be seen on the graph where the line crosses the y-axis. **(Recapping)**

EU: In certain contexts, the domain and range of the relationship that models the context are constrained by the context because certain values may not make sense in the situation.

- Will every pair of *x* and *y* values that make the equation true be a point on the graph?
- I saw many groups connect the points on the graph. Does every value make sense? Can you buy 1.5 cones? I hear people saying "no," so then what should the graph look like?
- The input values of a function are the domain while the output values are the range. **(Marking)** Is the domain of the function the same as the domain of the graph? Explain.

Application	If Joe's on the Beach reduces the cost of the ice cream cake to $17.99, how does this affect the equation we created? The graph? The table?
Summary	Let's look at our 3 representations—table, equation, and graph. How does each representation model the situation? What values stay the same? What changes?
Quick Write	In your own words, describe the meaning of domain.
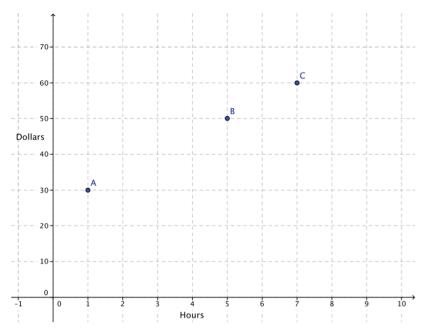
Support for students who are English Learners (EL):
1. Display images of an ice cream cake and ice cream cone so students identified as English Learners associate the words in the problem with the images.
2. Slow down the whole class discussion for students who are English Learners by asking other students to repeat key ideas and to put ideas in their own words.
3. Create a running list of ways in which the function models the problem situation and ask students who are English Learners to step back and notice similarities, differences, and/or patterns.

Name_____

Jose's Surfboard

Jose rents a surfboard for the day from a company that charges by the hour. The graph below shows the cost of renting a surfboard for different amounts of time.

1. What is the rental rate per hour for the surfboard? Explain your reasoning.

2. If the cost continues at this rate, calculate the cost of renting a surfboard for 12 hours. Show all work and explain your reasoning.

3. Calculate the number of hours Jose surfs if the rental cost is $150.00. Show all work and explain your reasoning.

4. Is the domain of the function the same as the domain of the problem situation? Explain your reasoning.
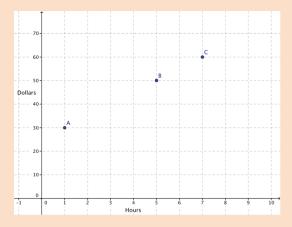
Jose's Surfboard

Rationale for Lesson: Continue developing understanding of creating functions and how the context affects possible solution values for the function. In this lesson, students begin with a graph and use the rate of change to answer questions.

Task 2: Jose's Surfboard
Jose rents a surfboard for the day from a company that charges by the hour. The graph below shows the cost of renting a surfboard for different amounts of time.

1. What is the rental rate per hour for the surfboard? Explain your reasoning.
2. If the cost continues at this rate, calculate the cost of renting a surfboard for 12 hours. Show all work and explain your reasoning.

See student paper for complete task.

Tennessee State Standards	A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
	F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP4 Model with mathematics. MP6 Attend to precision. MP7 Look for and make use of structure.
Essential Understandings	• The solution to an equation can be represented algebraically, graphically, in a table, and in a context. • The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values. • In certain contexts, the domain and range of the relationship that models the context are constrained by the context because certain values may not make sense in the situation.
Materials Needed	• Task. • Calculator (optional).

▶ **SET-UP PHASE**

Somebody please read the task aloud. Now take 5 minutes to work on the task before turning and talking to your partner(s).

▶ **EXPLORE PHASE**

Possible Student Pathways	Assessing Questions	Advancing Questions
Group can't get started.	What can you tell me about this point (pointing to [1, 30])? What does it mean in the context of this problem?	How is the cost changing from 0 to 1 hours? 1 to 2 hours?
Creates a table. Time / Cost 0 / 25 5 / 50 10 / 75 15 / 100	How did you determine the values in your table?	What patterns do you see in the table? How can you use these patterns to predict the cost of renting a surfboard for 12 hours?
Extends the line.	Can you explain your strategy? How do you know extending the line is a strategy that works?	How can you use the line to find the cost of a 12-hour rental? The number of hours you can rent for $150?
Writes a function to model the situation.	What do the terms in your function mean with respect to the context?	How can you use the function to make predictions? Is the domain of your function the same as the domain of the context?

▶ **SHARE, DISCUSS, AND ANALYZE PHASE**

EU: The solution to an equation can be represented algebraically, graphically, in a table, and in a context. Linear equations can be used to model real-world situations.
- Who can explain how to write a function to represent this situation?
- What do each of the points on the graph represent? Explain.
- How does your function represent the information on the graph symbolically?
- What does each term represent? *(The 5 represents the cost per hour and the 25 represents the one-time fee.)*
- Right. The 5 represents the cost per hour, or the rate of change, and the 25 represents the y-intercept, or the cost for zero hours. **(Revoicing)**

EU: The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values.
- Let's take a look at how different groups determined the number of hours Jose rented the surfboard.
- I notice several groups represented the problem in different ways. Who can explain how to use the graph to tell how much Jose has to pay to rent his surfboard?
- How does the graph show the hourly rate that Jose is paying for his surfboard rental?
- How does this appear in the table and equation?
- We know that linear functions have constant rates of change. How does the graph reflect this characteristic? The equation? How will it appear in a table of values for this function?
- Can somebody summarize the key points in this explanation?

EU: In certain contexts, the domain and range of the relationship that models the context are constrained by the context because certain values may not make sense in the situation.
- What does domain mean? What is the domain of this problem situation?
- What values can you substitute for x into the equation? Do all of these values make sense?
- Discuss with your partners whether you feel decimal values, negative numbers, and all positive numbers should be included. *(Since x represents time and the surfboard is rented by the hour, it doesn't make sense to rent for negative dollars or negative time.) (Since they rent by the hour, you can't rent for part of an hour. Only whole numbers of hours make sense.)*
- What do others think? Can you rent for part of an hour? *(If you keep the surfboard for too long they probably charge you for another hour. So, 2 hours and 15 minutes would cost as much as 3 hours.)*
- I'm hearing groups say that the cost is defined for any positive amount of time, but that it jumps up all at once after certain amounts of time instead of increasing at a constant rate. Other groups argue that the context only makes sense for whole number values of time. Each of these interpretations has a graph that is not continuous, because the curve (line) is not smooth and unbroken.
- We agree that the domain of the problem situation is different from the domain of the equation, but is it possible that we may disagree on the domain of the problem situation?

Application	If the cost of renting a surfboard is $7 per hour, how will the graph and equation change? How will they stay the same?
Summary	Where do the slope and the y-intercept appear in the graph? What do they mean in the problem situation?
Quick Write	Describe in as many ways as possible how you know that (2, 35) is a solution to the equation.
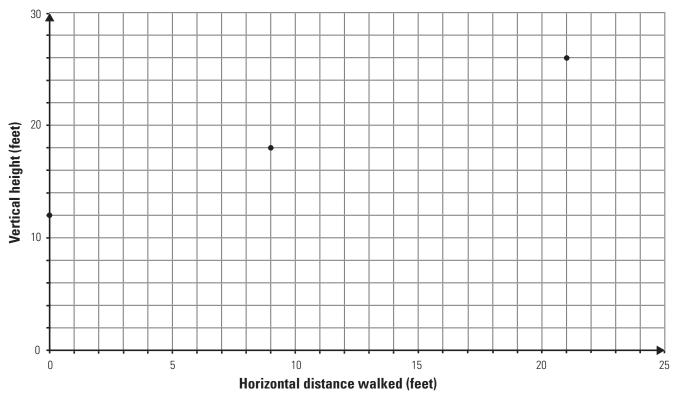
Support for students who are English Learners (EL):
1. Bring in or display an image of a surfboard so students identified as English Learners associate the word in the problem with the image.
2. Slow down the whole class discussion for students who are English Learners by asking other students to repeat key ideas and to put ideas in their own words.
3. Create a running list of connections between the equation and the graph and ask students who are English Learners to step back and notice similarities, differences, and/or patterns. Create a classroom artifact showing the meaning of rate of change and how it may appear in the graph/table/equation.

Name_____

Ocoee Sand Dunes

A state park near Jose's hotel has very high sand dunes that increase in elevation at a constant rate. The graph shows Jose's elevation at various points as he climbs one of the sand dunes.

1. Explain how you know that the function $h(x) = \dfrac{2}{3} x + 12$ correctly models this problem situation.

2. A park ranger says that for every 5 feet Jose travels in a horizontal direction, he climbs 4 feet vertically. Do you agree or disagree with this statement? Justify your answer mathematically.

3. A student claims that the domain of this problem situation is [0, 9, 21] and the range is [12, 18, 26]. Do you agree or disagree? Explain your reasoning.
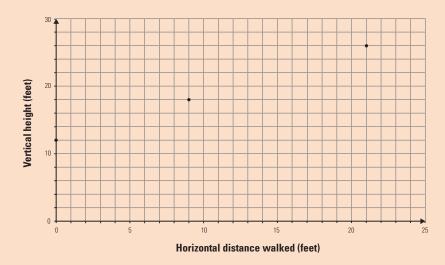
Ocoee Sand Dunes

Rationale for Lesson: Continue developing an understanding of domain and range, as well as the relationship between the graphical and algebraic representations of a function. In this lesson, students are given the correct function and have to describe why this function models the situation.

Task 3: Ocoee Sand Dunes

A state park near Jose's hotel has very high sand dunes that increase in elevation at a constant rate. The graph shows Jose's elevation at various points as he climbs one of the sand dunes.

Horizontal distance walked (feet)

1. Explain how you know that the function $h(x) = \dfrac{2}{3}x + 12$ correctly models this problem situation.

See student paper for complete task.

Tennessee State Standards	A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
	F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

LESSON GUIDE 3

Tennessee State Standards	MP1 Make sense of problems and persevere in solving them. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP6 Attend to precision. MP7 Look for and make use of structure.
Essential Understandings	• The solution to an equation can be represented algebraically, graphically, in a table, and in a context. • The graph of a linear relationship is a line that models the relationship between the variables in the context. The coordinates of the points on the line form the solution set for the associated linear equation. • In certain contexts, the domain and range of the relationship that models the context are constrained by the context because certain values may not make sense in the situation. • The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values.
Materials Needed	• Task.

▶ **SET-UP PHASE**

Somebody please read the task aloud. The pictures of sand dunes that I displayed will help you visualize the problem. Would somebody please restate question 2 in their own words? Now take 5 minutes to work on the task before turning and talking to your partner.

▶ **EXPLORE PHASE**

Possible Student Pathways	Assessing Questions	Advancing Questions
Tests the function by substituting values.	Can you explain how substitution lets you know whether this is the correct model?	How can you tell whether the model is correct by analyzing the graph?
Uses the slope and the *y*-intercept of the graph to test the model.	Where do you see the $\frac{2}{3}$ and the 12 on the graph? How does this let you know that the model is correct?	How can you use this method to tell whether the park ranger's assertion is correct?
Creates a table of values to test the model.	Why are you creating a table of values? How will that allow you to test the model?	What patterns do you see in the table that let you know the function is correct?
Students finish early.	How can you tell from the graph that the function is correct? Where is the rate of change and the slope in your graph and function?	Given any two points on a graph, say [5, 10] and [15, 21] that I plotted on your grid, can you determine the equation of the line that passes through these points?

▶ **SHARE, DISCUSS, AND ANALYZE PHASE**

EU: The solution to an equation can be represented algebraically, graphically, in a table, and in a context.
- I saw several groups testing points and several groups using the structure of the graph to determine whether $h(x) = \frac{2}{3}x + 12$ is the correct function. Let's compare both of these methods.
- Someone explain how you tested points and then I'm going to ask somebody to summarize this method in your own words.
- What does the point [3,14] mean in this problem?
- What does it mean if a point satisfies the equation?
- How can you use the graph to help test whether the function is correct?
- Are the coordinates shown on the graph the only solutions to the equation?

EU: The graph of a linear relationship is a line that models the relationship between the variables in the context and its coordinates form the solution set for the associated linear equation.
- Why do all of the solutions fall in a line?
- In other words, why is this function linear? *(The graph is a straight line, which means it increases at the same rate. The equation is in the form y = mx + b. The table of values increases at a constant rate.)*
- I'm hearing rate of change in each of these descriptions. The rate of change is constant in linear functions y = mx + b, where m is the rate of change. The graph is a line increasing at the same rate and the y-values increase at the same rate for every x increase of 1. **(Recapping)**

EU: In certain contexts, the domain and range of the relationship that models the context are constrained by the context because certain values may not make sense in the situation.
- What does the graph tell us about the increase of the sand dunes? Explain.
- I heard a group say that the slope in the equation should be $\frac{4}{6}$. Can someone explain to us whether they agree or disagree with that group?
- How did you use this to test the park ranger's assertion? Was he correct? Explain.
- Some groups agreed and some groups disagreed with the statement in question 3. Who can explain why they agree or disagree with this statement?

EU: The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values.
- Why is the graph showing individual points rather than a solid line? Are these the only solutions to the function?
- What values make sense in this problem context? Where will other solutions appear? *(All of the values along the line make sense.)*
- I hear groups noting the points should be connected and that the function is continuous. **(Marking)**
- Why should we connect the points in this graph and NOT connect the points in the previous task?

Application	If Jose walks 100 feet in a horizontal direction, what height will he reach? Explain how you know.
Summary	How was this problem the same/different from the surfboard problem? (Reference artifact from previous lesson.)
Quick Write	Write a sentence explaining how a graph can help you write an equation for a linear function.

Name_____

More Sand Dunes

Jose hikes a different dune the following week. He starts at sea level.

1. Determine the equation that represents this section of the new dune shown on the graph. Explain where the rate of change appears in the graph and the equation.

Ocoee Sand Dune Hike

Horizontal axis: **Horizontal Distance Walked (feet)**
Vertical axis: **Vertical Height (feet)**
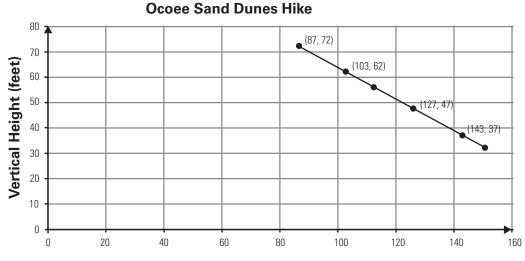
2. After Jose reaches a height of 72 feet, the next portion of his hike can be modeled with the graph below. Calculate the rate of change for his hike over this interval. Explain what the rate of change represents in this problem situation.

Ocoee Sand Dune Hike

Points: (27, 72), (57, 72), (87, 72)

Horizontal axis: **Horizontal Distance Walked (feet)**
Vertical axis: **Vertical Height (feet)**

3. The sketch below shows part of the final portion of Jose's hike. Explain how you know that the equation $y = -\frac{5}{8}x + 126.375$ correctly represents this portion of the hike.

Ocoee Sand Dunes Hike

Horizontal axis: **Horizontal Distance Walked (feet)**, marked 0, 20, 40, 60, 80, 100, 120, 140, 160.
Vertical axis: **Vertical Height (feet)**, marked 0, 10, 20, 30, 40, 50, 60, 70, 80.

Plotted points: (87, 72), (103, 62), (127, 47), (143, 37).
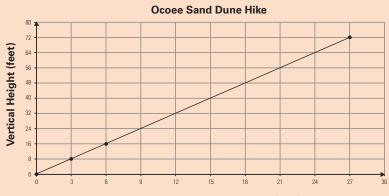
More Sand Dunes

Rationale for Lesson: Solidify understanding of rates of change, and the relationship between rates of change and linear equations. In this lesson, students analyze specific intervals on a graph and solidify a method for calculating rate of change through repeated reasoning.
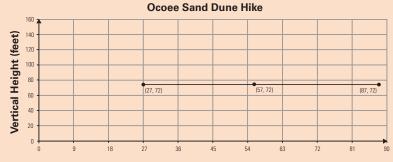
Task 4: More Sand Dunes
Jose hikes a different dune the following week. He starts at sea level.
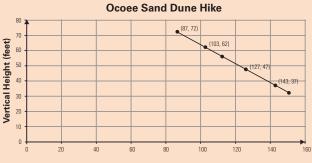
1. Determine the equation that represents this section of the new dune shown on the graph. Explain where the rate of change appears in the graph and the equation.

Ocoee Sand Dune Hike

Ocoee Sand Dune Hike

Ocoee Sand Dune Hike

2. After Jose reaches a height of 72 feet, the next portion of his hike can be modeled with the graph below. Calculate the rate of change for his hike over this interval. Explain what the rate of change represents in this problem situation.

3. The sketch below shows part of the final portion of Jose's hike. Explain how you know that the equation $y = -\frac{5}{8}x + 126.375$ correctly represents this portion of the hike.

Tennessee State Standards	A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP6 Attend to precision. MP7 Look for and make use of structure. MP8 Look for and express regularity in repeated reasoning.	
Essential Understandings	• The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values. • The graph of a linear relationship is a line that establishes the relationship between the variables in the context and its coordinates form the solution set to the associated linear equation.	
Materials Needed	• Task. • Calculator (optional).	

▶ **SET-UP PHASE**

Somebody please read the task aloud. Now take 5 minutes to work on the task before turning and talking to your partner.

▶ **EXPLORE PHASE**

Possible Student Pathways	Assessing Questions	Advancing Questions
Students use the slope and *y*-intercept to create the equation in question 1.	Can you tell me about your calculations? How do you know they represent the problem?	How can you be certain that this equation represents the entire solution set?
Students confirm that the equation is correct by confirming that the slope and the *y*-intercept are the same.	How do you know the slope and the *y*-intercept are the same? Why does this guarantee that it is the only unique equation to model this relationship?	How can you confirm that the equation is correct for the other points? Can you think of another way to confirm the equation?
Students confirm that the equation is correct by substituting points into the equation.	Can you tell me why your method works?	Do you have to substitute all points into the equation? How do you know? Can you think of another way to confirm the equation?
Students confirm that the equation is correct by extending the line.	Can you explain your method to me? What key points are you trying to identify?	Extending the line may not be exact. How can you confirm algebraically that the equation is correct?
Finishes early.	Can you summarize your strategy for each of these problems?	Can you summarize how each of these problems was the same/different?

▶ **SHARE, DISCUSS, AND ANALYZE PHASE**

EU: The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values.

- In question 1, who can tell me the relationship between the two variables shown on the graph?
- How does the height change as the horizontal distance changes? *(For a horizontal increase of 3 the vertical change is 8. This continues for the whole line.)*
- Would someone please restate this idea in his or her own words, explaining why the rate of change is the same? Come up and point to where this appears in the graph.
- How is the rate of change used to write the equation?
- I see that not all groups used the same two points to calculate the rate of change in question 1. Can you use any two points? Why is that? **(Challenging)**
- Someone said that the rate of change shown in question 2 is a constant. Could someone else please explain why that person said that? *(The rate of change is 0, which is a constant.)*
- You all just said that the rate of change for the graph in question 2 is 0. How would you describe in words Jose's hike at that point in time?
- What is the rate of change for the third portion of his hike? Is it constant? How do you know? *(It is constant, but it is negative.)*
- So I'm hearing that whether the line increases, decreases, or is horizontal, the rate of change is constant. **(Recapping)**

EU: The graph of a linear relationship is a line that establishes the relationship between the variables in the context and its coordinates form the solution set to the associated linear equation.

- Where does the rate of change appear in the function that represents the third portion of his hike?
- Can somebody come up and explain where it appears on the graph?
- I notice you extended your line to check that the *y*-intercept is 126. Who can show us another way that does not involve extending the line? How is the rate of change used in this method? *(I can substitute a value for x and the y-value is the output. This should correspond to the x and y coordinates for each point on the graph.)*
- I noticed that when you substituted an x-value into the equation, you checked to see if the output, or the y-value, matched. **(Marking)** How many points do you need to check in order to verify that this is the correct equation? How do you know?
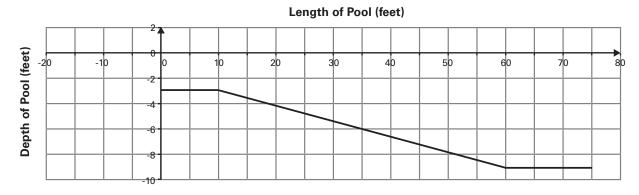
Application	What is the rate of change over the interval from [143, 37] to [167, 22]?
Summary	Let's put together what we learned over the last several lessons about rates of change. Would someone please restate what the differences are between positive, negative, and zero rates of change? Where does rate of change appear in the graph and the equation?
Quick Write	Write a sentence explaining how the graph of a linear function shows that the rate of change is constant.

Name_____

Swimming Pool Depth

The graph below shows the depth of a swimming pool over its 75-foot length.

Length of Pool (feet)

1. Describe how the depth of the pool is changing with respect to the length of the pool.

2. Calculate the rate of change of the depth over each of the intervals of length below. Describe the meaning of each rate of change.
   A. [0, 10]
   B. [10, 60]
   C. [60, 75]

3. The average rate of change of the depth of the pool over the interval [0, 75] is different than the average rate of change for [10, 60]. Explain why this is the case.

4. **Extension**: Determine the function that represents the depth of the pool, y, over the following intervals of length, *x*.
   A. [0, 10]
   B. [10, 60]
   C. [60, 75]

Swimming Pool Depth
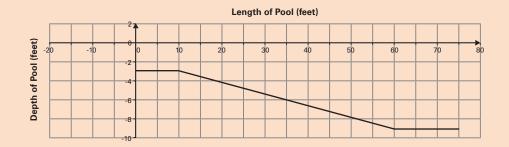
Rationale for Lesson: Continue developing an understanding of zero, negative, and positive slopes in the context of piecewise functions. In this lesson, interval notation is formally introduced and students encounter average rate of change for the first time.

Task 5: Swimming Pool Depth
The graph below shows the depth of a swimming pool over its 75-foot length.

1. Describe how the depth of the pool is changing with respect to the length of the pool.

Length of Pool (feet)

Depth of Pool (feet)

2. Calculate the rate of change of the depth over each of the intervals of length below. Describe the meaning of each rate of change.
    A. [0, 10]
    B. [10, 60]
    C. [60, 75]
3. The average rate of change of the depth of the pool over the interval [0, 75] is different than the average rate of change for [10, 60]. Explain why this is the case.

See student paper for complete task.

Tennessee State Standards	F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
	F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

LESSON GUIDE 5	**Standards for Mathematical Practice**	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP6 Attend to precision. MP7 Look for and make use of structure.
	Essential Understandings	• The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values. • The average rate of change is the ratio of the change in the dependent variable over a specified interval in the domain to the change in the independent variable over the same interval.
	Materials Needed	• Task. • Calculator (optional).

▶ **SET-UP PHASE**

Everybody please read the task silently *(wait a few moments)*. Would someone please use his or her own words to summarize what this task is asking you to do? We've worked on rate of change, but now what do you think is meant by average rate of change? How would this be different from rate of change? What is different about this function that may require new language about change? Discuss for a few minutes with your partner before we regroup and continue talking about this task.

▶ **EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)**

Possible Student Pathways	Assessing Questions	Advancing Questions
Group can't get started.	What does rate of change mean? Point to the intervals that you are considering. How is the graph the same/different over each interval?	How are the y-values changing for each x-interval? Are they going up, down, or staying the same? How do you know?
Calculates the rate of change over each interval, adds them together, and divides by 3.	Can you explain your method? When will this method work?	Can you determine a method so that you don't have to calculate the rate of change over each interval?
Uses a formula to determine the average rates of change.	What does the average rate of change mean? How is this shown on the graph?	Can you sketch a different graph that has the same average rate of change?
Group finishes early.	How did you determine the equations? Why are these two equations constant values?	Can you make a generalization about how to determine the equation over an interval when the *y*-intercept is not shown?

▶ **SHARE, DISCUSS, AND ANALYZE PHASE**

EU: The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values.

- Would someone please explain the meaning of the graph in the context of this problem situation?
- Somebody else use that group's explanation to explain why does the graph look flat for this interval (points) and why it then suddenly starts to descend? *(The depth decreases and then it remains the same for this portion, and then it decreases again.)*
- We call functions that behave differently like this from one interval to the next qualitative functions. **(Marking)**
- Let's look at the rates of change that different groups calculated. Why is the rate of change over this horizontal interval and this other horizontal interval the same even though they're in different locations on the graph? *(The y-values stay the same over these intervals, but the x-values change.)*
- You are saying that since the y-values stay the same, the horizontal intervals have a zero rate of change. **(Marking)** Will all horizontal lines have a zero rate of change? Why?
- We see that it's zero using a slope equation, but who can add on to tell us based on the problem situation why the rate of change must be zero?
- Are the rates of change over each interval constant values? Why or why not?

EU: The average rate of change is the ratio of the change in the dependent variable over a specified interval in the domain to the change in the independent variable over the same interval.

- We've considered three different intervals here, each with a unique rate of change. What does average rate of change mean? Who can tell us about situations in your life where you have determined an average, and then explain what an average means?
- How do these examples that everybody has given relate to this problem? What does the average rate of change represent here?
- How did you determine whether the average rates of change would be the same or different?
- Can somebody summarize a method that can be used to calculate average rate of change? *(The average rate of change is the change in y over the entire interval. The rate of change may be different for different parts of it, but you determine the total change in y divided by the total change in x.)*
- If a function is linear, will the average rate of change be the same for all intervals? **(Challenging)** How do you know?
- Is there an average rate of change formula that we can use? Explain. *(We can calculate the total change in y and divide by the total change in x.)*
- So we can use this formula (f(b) - f(a))/(b - a), which is the change in y divided by the change in x. **(Recapping)**

Application	Determine the rate of change if the depth drops from 3 feet to a depth of 11 feet over the interval [10, 55].
Summary	Would someone please summarize for us what we mean by average rate of change? How is this different from just calculating the rate of change?
Quick Write	Describe what you learned about average rate of change. Then write one question that you still have about today's lesson.

Support for students who are English Learners (EL):
1. Display images of pools, showing the changes in depth, so students identified as English Learners associate the word in the problem with the image.
2. Slow down the whole class discussion for students who are English Learners by asking other students to repeat key ideas and to put ideas in their own words.

Name_____

Walking

Two sisters, Bonita and Rosie, took a walk at the same time but from different starting points. They described their walks below.

Bonita	**Rosie**
I started at home. I left the house and walked at a constant rate of 0.3 miles every 10 minutes away from the house. I walked for 15 minutes and then stopped.	I started 1 mile from our house and walked toward home. For the first 4 minutes, I walked at a constant rate until I was 0.8 miles from home. Then I stopped for 4 minutes to talk to a friend. After that, I continued walking toward home at a constant rate of 0.1 miles per minute until I arrived at the door.

1. Sketch a graph of each girl's walk on the coordinate plane below. Label your lines and explain why you believe your lines represent their journeys.

Distance from Home (miles)

Time (minutes)

2. Decide whether you agree or disagree with each of the following statements. Justify your answer mathematically.
    A. Bonita is walking faster than Rosie for the interval [0, 4].
    B. Rosie's average rate of change was greater than Bonita's the first 15 minutes.
    C. The rate of change is the same over at least one interval of their walk.
    D. The girls meet each other at some point during their walks.

Extension: Determine the linear function that represents each girl's walk over the intervals [0, 4], [4, 8], and [8, 15]. Explain how you know the function models each girl's journey.

Walking
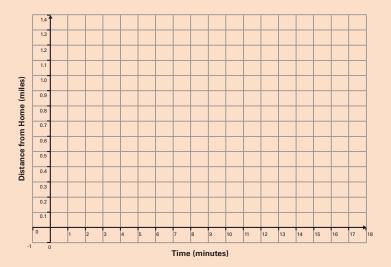
Rationale for Lesson: Continue developing an understanding of rate of average change. Students draw and interpret a graph from a written context.

Task 6: Walking

Two sisters, Bonita and Rosie, took a walk at the same time but from different starting points. They described their walks below.

Bonita	Rosie
I started at home. I left the house and walked at a constant rate of 0.3 miles every 10 minutes away from the house. I walked for 15 minutes and then stopped.	I started 1 mile from our house and walked toward home. For the first 4 minutes, I walked at a constant rate until I was 0.8 miles from home. Then I stopped for 4 minutes to talk to a friend. After that, I continued walking toward home at a constant rate of 0.1 miles per minute until I arrived at the door.

1. Sketch a graph of each girl's walk on the coordinate plane below. Label your lines and explain why you believe your lines represent their journeys.

2. Decide whether you agree or disagree with each of the following statements. Justify your answer mathematically.
   A. Bonita is walking faster than Rosie for the interval [0, 4].
   B. Rosie's average rate of change was greater than Bonita's the first 15 minutes.
   C. The rate of change is the same over at least one interval of their walk.
   D. The girls meet each other at some point during their walks.

Extension: Determine the linear function that represents each girl's walk over the intervals [0, 4], [4, 8], and [8, 15]. Explain how you know the function models each girl's journey.

Tennesee State Standards	A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Tennessee State Standards	F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
	F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP6 Attend to precision.	
Essential Understandings	• The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values. • The average rate of change is the ratio of the change in the dependent variable over a specified interval in the domain to the change in the independent variable over the same interval.	
Materials Needed	• Task. • Extra graph paper. • Calculator (optional).	

▶ **SET-UP PHASE**

Everybody please read the task silently *(wait a few moments)*. Would someone please use his or her own words to summarize the walk described by Bonita and Rosie? What does it mean again to walk at a constant rate? Now take 5 minutes to work on the task before turning and talking to your partner.

▶ **EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)**

Possible Student Pathways	Assessing Questions	Advancing Questions
Group can't get started.	Where is each girl starting? Can you point to the location on the coordinate plane?	Can you locate those points on the graph? How quickly is each girl walking? For how long?
Determines the greater average rate of change by calculating which has the greater difference in *y*-values over the interval [0, 15].	Describe your method to me. How does this relate to what is happening in the problem situation?	Can you generalize this method into a formula?
Uses a formula to calculate average rate of change.	What do the numbers in your formula represent in the context of the problem?	Why is Rosie's rate of change negative while Bonita's is positive? Does that mean that Bonita is always traveling faster?
Finishes early.	How did you determine the equation of the lines over each interval? Over which intervals is the equation constant? Why?	Can you generalize a method for determining the equation over a linear interval if you know any two points?

▶ **SHARE, DISCUSS, AND ANALYZE PHASE**

EU: The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values.

- Would someone please explain why you modeled the situation with this graph?
- Somebody else use that group's explanation to explain why the one line is increasing while the other is not. Please point to the intervals as you describe them and explain how it relates to the problem.
- What does a negative rate of change represent in this context? What does a zero rate of change represent?
- How did different groups determine which girl was walking at a faster rate over different intervals?
- What two variables are changing? Why is the change in *y* significant? How does this relate to the graph?
- We discussed this in a previous task, but somebody remind us why the rate of change over this horizontal interval (points) and this other horizontal interval (points) are the same even though they're in different locations on the graph. *(They are the same because there is no change in distance over time.)*
- So Bonita and Rosie are traveling at the same rate of zero when they stopped walking. **(Marking)** Are there any other intervals over which they are traveling at the same rate? How do you know from the graph? How do you know from the equations you wrote?

EU: The average rate of change is the ratio of the change in the dependent variable over a specified interval in the domain to the change in the independent variable over the same interval.

- We've considered different intervals here, each with a unique rate of change. Who can remind us from the last lesson what the average rate of change means?
- How did your group determine the average rates of change?
- Can somebody revoice how this group defined average rate of change and how they used their understanding to determine which girl had the greater average rate of change?
- If a function is linear, will the average rate of change over an interval be the same as the rate of change for any two points? Why or why not? *(Yes, the change is constant throughout, so it doesn't matter what interval you choose.)*
- That's an important point. The rate of change for a linear function is constant. Rate of change and average rate of change will be the same for all linear functions. We are concerned with average rate of change only when the function has different rates of change for different intervals. **(Marking)**

Application	Sketch the graph of a walker who takes a different path but has the same average rate of change as Bonita. Explain how you know the average rate of change is the same.
Summary	How is this problem the same/different from the Swimming Pool Task?
Quick Write	No quick write for students.

Support for students who are English Learners (EL):
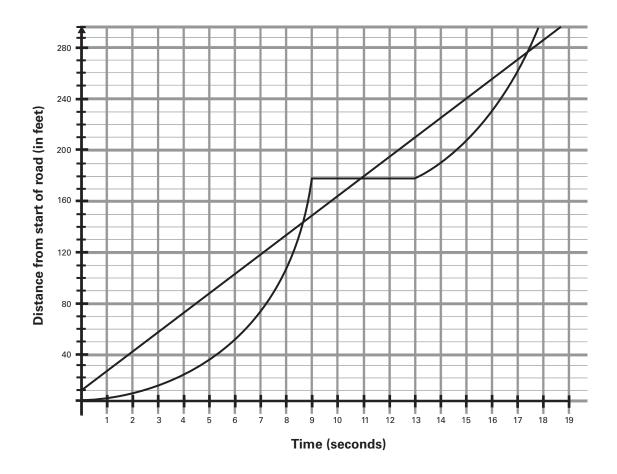
1. During the Set-Up, have ELs discuss the descriptions of the walkers. Translate specific words that are unclear and co-create a diagram that shows the paths that the girls are taking.

2. Slow down the whole class discussion for students who are ELs by asking other students to repeat key ideas and to put ideas in their own words.

Name_____

Bike and Truck

A bicycle traveling at a steady rate and a truck are moving along a road in the same direction. The graph below shows their positions as a function of time. Let B(t) represent the bicycle's distance and K(t) represent the truck's distance.

1. Label the graphs appropriately with B(t) and K(t). Explain how you made your decision.

2. Describe the movement of the truck. Explain how you used the values of t and K(t) to make decisions about your description.

3. Which vehicle was first to reach 300 feet from the start of the road? How can you use the domain and/ or range to determine which vehicle was the first to reach 300 feet? Explain your reasoning in words.

4. Jack claims that the average rate of change for both the bicycle and the truck was the same in the first 17.5 seconds of travel. Explain why you agree or disagree with Jack.

Bike and Truck

Rationale for Lesson: Students use the language of functions and function notation to interpret and describe the position from the start of a road of a bike and a truck, which is presented graphically.

Task 7: Bike and Truck
A bicycle traveling at a steady rate and a truck are moving along a road in the same direction. The graph below shows their positions as a function of time. Let B(t) represent the bicycle's distance and K(t) represent the truck's distance.

1. Label the graphs appropriately with B(t) and K(t). Explain how you made your decision.
2. Describe the movement of the truck. Explain how you used the values of t and K(t) to make decisions about your description.
3. Which vehicle was first to reach 300 feet from the start of the road? How can you use the domain and/or range to determine which vehicle was the first to reach 300 feet? Explain your reasoning in words.
4. Jack claims that the average rate of change for both the bicycle and the truck was the same in the first 17.5 seconds of travel. Explain why you agree or disagree with Jack.

Tennessee State Standards	F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
	F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
Standards for Mathematical Practice		MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP6 Attend to precision.
Essential Understandings		• The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values. • The average rate of change is the ratio of the change in the dependent variable over a specified interval in the domain to the change in the independent variable over the same interval.
Materials Needed		• Task. • Calculator (optional).

▶ **SET-UP PHASE**

Read the information at the top of the page and look at the graph. Analyze the graph individually to determine which function represents the bike and which function represents the truck before working with partners.

▶ **EXPLORE PHASE**

Possible Student Pathways	Assessing Questions	Advancing Questions
Students cannot interpret values on the graph in terms of the context.	If the bike and truck are in the same place at the same time, how is that visible on the graph?	How can we tell over which intervals the truck was in the lead?
Relates the domain of the function to its graph.	Why do these functions have the same range?	What might be the domain and range of the graph of a vehicle that moved faster than both the bike and the truck?
Interprets rates of change in context.	Over which intervals was the truck moving fastest? How can you determine this information from the graph?	Can the one (bike or truck) with the slower speed be ahead? Explain. What is happening on the graph and in context at these points (or intervals)?
Group finishes early.	How are the variable quantities changing in each of these functions? What does this relationship mean in context and how is it visible in the graph?	Are there any intervals where the bike and truck appear to be moving at about the same speed? Explain your reasoning.

▶ SHARE, DISCUSS, AND ANALYZE PHASE

EU: The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values.

- How would you describe the movement of the bike and the truck?
- What do you notice about the rates of change in this graph?
- Do either of these functions ever have a negative rate of change? What would a negative rate of change look like? What would it mean in context? **(Challenging)**
- What do the points of intersection represent on this graph? What is true about the bike and truck's speed at and near these points? *(The intersection points show where they have traveled the same distance at the same time.)*
- So this point represents when their distance is the same and their time is the same **(Marking)** Does this mean they have to be at the same place? **(Challenging)**
- What two variables are being related in this problem? What does the rate of change tell you about the relationship between these variables?
- Does the bike or the truck cover more distance for each one-second increase in time over the course of the race? Is this true over every interval of the race?

EU: The average rate of change is the ratio of the change in the dependent variable over a specified interval in the domain to the change in the independent variable over the same interval.

- What is meant by average rate of change?
- Does the bike or the truck have a greater average rate of change over the 300 feet indicated in the graph? How do you know?
- Do either of these functions have a constant rate of change? How can you determine this from the graph?
- Over which intervals is the bike's average rate of change greater than truck's? Is the bike in the lead over these intervals? Explain.
- Who can summarize how average rate of change is calculated? *(You determine who traveled the greatest distance by subtracting and then dividing by the amount of time).*
- You can see that the change is different for various intervals. We can make sense of their change by calculating the average rate of change, which is the change in *y*-values divided by the change in x-values. **(Recapping)**

Application	How far from the road are the truck and bike after 9 seconds have passed?
Summary	We learned today that only linear functions have constant rates of change Would someone please summarize the difference between the rates of change for the bike and the truck?
Quick Write	Would you describe your understanding of average rates of change as being fuzzy or clear?
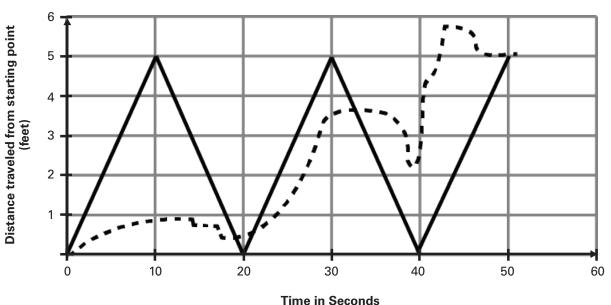
Name_____

Sandpiper vs. the Sand Crab

Sandpipers are beach birds that love to eat sand crabs. Sandpipers run back and forth along the sand at the edge of the water. The graph shows the path of a sandpiper and the path of a sand crab as it runs toward its hole in the sand. The solid line represents the sandpiper's path.

Sandpiper vs the Sand Crab

1. Calculate the average rate of change for the sandpiper as it moves across the sand during each interval:
   A.   [0, 10]
   B.   [0, 20]
   C.   [20, 30]
   D.   [20, 40]
   E.   [40, 50]

   Generalize a method for determining average rate of change. Describe any patterns that you notice.

2. A hermit crab and a lobster join the sandpiper and the sand crab on the beach. Sketch the graph of each with the following conditions:

   A.   The hermit crab has the same average rate of change as the sand crab.
   B.   The lobster has a greater average rate of change than the sand crab.
        Explain your reasoning for each drawing.
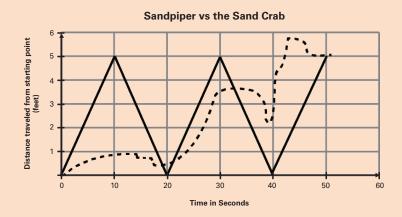
Sandpiper vs. the Sand Crab

Rationale for Lesson: Solidify an understanding of average rate of change. In this lesson, students interpret two curves on a coordinate plane, then draw and interpret graphs with average rate constraints from a written context.

Task 8: Sandpiper vs. the Sand Crab
Sandpipers are beach birds that love to eat sand crabs. Sandpipers run back and forth along the sand at the edge of the water. The graph shows the path of a sandpiper and the path of a sand crab as it runs toward its hole in the sand. The solid line represents the sandpiper's path.

Sandpiper vs the Sand Crab

1. Calculate the average rate of change for the sandpiper as it moves across the sand during each interval:
   A. [0,10]
   B. [0,20]
   C. [20,30]
   D. [20,40]
   E. [40,50]

Generalize a method for determining average rate of change. Describe any patterns that you notice.

See student paper for complete task.

Tennessee State Standards	F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP6 Attend to precision. MP8 Look for and express regularity in repeated reasoning.	

<table>
<tr><td rowspan="2">**LESSON GUIDE 8**</td><td>**Essential Understandings**</td><td>• The average rate of change is the ratio of the change in the dependent variable over a specified interval in the domain to the change in the independent variable over the same interval.</td></tr>
<tr><td>**Materials Needed**</td><td>• Task.</td></tr>
</table>

▶ **SET-UP PHASE**

Everybody please read the task silently *(wait a few moments)*. Take a few moments on your own to make sense of the graph. Remember to reference the word wall for definitions of rate of change and average rate of change. Has anybody ever seen a sandpiper or sand crab? Can you describe them? I brought in a picture so everybody can visualize them.

Now take 5 minutes to work on the task before turning and talking to your partner.

▶ **EXPLORE PHASE**

Possible Student Pathways	Assessing Questions	Advancing Questions
Determines the rate of change over each interval.	Describe your method to me. How does this relate to what is happening in the problem situation?	Can you come up with a more efficient method using just two points?
Uses a formula to calculate average rate of change.	What do the numbers in your formula represent in the context of the problem?	What would the average rate of change be for [0, 40]? Can you use what you know about average rate of change to calculate this simply by observation (without using the formula)? Explain.
Sketches a linear function for the hermit crab.	Can you explain the movement of the hermit crab?	What if he did not travel at a constant speed? Can you sketch his movement?
Finishes early.	How did you determine the average rate of change? Describe your method for sketching the lines.	If you calculate the equation for each of the sandpiper intervals from 0 to 40, what patterns will you notice in the equations? What does this have to do with average rate of change?

▶ **SHARE, DISCUSS, AND ANALYZE PHASE**

EU: The average rate of change is the ratio of the change in the dependent variable over a specified interval in the domain to the change in the independent variable over the same interval.

- Would someone please explain how your group determined the average rates of change?
- Explain what two variables are changing. Why is the change in *y* significant? How does this relate to the graph?
- Somebody else use that group's explanation to explain how you can use the formula for certain intervals, but then how you know the average rate of change is zero for others? Please point to the intervals as you describe them and explain how it relates to the problem. *(We're looking at the distance traveled from the starting point. This results in an average rate of change of zero because for the first 10 seconds it's moving away, but then the next 10 seconds it's moving back at the same rate.)*
- I'm hearing you say that the rate of change is the same magnitude, but the sign is different. **(Marking)** Why is the average rate of change then zero?
- Somebody summarize what a positive/negative rate of change represents in this context? What does a zero rate of change represent?
- Somebody show us how you used all of this information to sketch the graphs of the hermit crab and the lobster. In your explanation, explain what may be different but what must be the same.
- Will somebody else summarize what was just said – why did the hermit crab end at the same location, but it could have taken a much different path to get there?
- Let's chart the mathematical truths that we have learned about average rate of change.

Application	Sketch the graph of a function over the interval [0, 10] that has an average rate of change of zero.
Summary	How is rate of change the same/different from average rate of change?
Quick Write	No quick write for students.

Support for students who are English Learners (EL):
1. During the Set-Up, have ELs discuss the context of the problem. Make pictures available to make the context more concrete and translate specific words that are unclear and co-create a list of terms.

Progressions

Appendix N
Modeling, High School

Progressions for the Tennesee State Standards in Mathematics (draft)

4 July 2013

Modeling, High School

Introduction

Mathematical models describe situations in the world, to the surprise of many. Albert Einstein wondered, "How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?"[•] This points to the basic reason to model with mathematics and statistics: to understand reality. Reality might be described by a law of nature such as that governing the motion of an object dropped from a height above the ground[A-CED.2] or in terms of the height above the ground of a person riding a Ferris wheel,[F-TF.5] the unemployment rate,[N-Q.1] how people's heights vary,[S-ID.1] a risk factor for a disease,[S-ID.5] the effectiveness of a medical treatment,[S-ID.5] or the amount of money in a savings account to which periodic additions are made.[A-SSE.4] On a more sophisticated level, modeling the spread of an epidemic, assessing the security of a computer password, understanding cyclic populations of predator and prey in an ecosystem, finding an orbit for a communications satellite that keeps it always over the same spot, estimating how large an area of solar panels would be enough to power a city of a given size, understanding how global positioning systems (GPSs) work, estimating how long it would take to get to the nearest star—all can be done using mathematical modeling. A survey of how mathematics has impacted recent breakthroughs can be found in Fueling Innovation and Discovery: The Mathematical Sciences in the 21st Century.[•]

   Mathematical modeling is fundamental to how mathematics is used in medicine, engineering, ecology, weather forecasting, oil exploration, finance and economics, business and marketing, climate modeling, designing search engines, understanding social networks, public key cryptography and cybersecurity, the space program, astronomy and cosmology, biology and genetics, criminology, using genetics to reconstruct how early humans spread over the planet, in testing and designing new drugs, in compressing images (JPEG) and music (MP3), in creating the algorithms that cell phones use to communicate, to optimize air traffic control and schedule flights, to design cars and wind turbines, to recommend which books (Amazon), music (Pandora) and movies (Netflix) an individual might like based on other things they rated highly. The range of careers for

- [•] January 27, 1921, address to the Prussian Academy of Science, Berlin.

- [•] 1960, "The Unreasonable Effectiveness of Mathematics," Communications in Pure and Applied Mathematics.

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

S-ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

S-ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

A-SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

- [•] This report was published in 2012 by the National Academies Press and can be read online at `http://www.nap.edu/catalog.php?record_id=13373.`

which mathematical and statistical modeling are good preparation has expanded substantially in recent years, and the list continues to grow.

Mathematical and statistical models of real world situations range in complexity from objects or drawings that represent addition and subtraction situations[K.OA.2] to systems of equations that describe behaviors of natural phenomena such as fluid flow or the paths of ballistic missiles. Sometimes models give rather complete information about the situation. For example, writing total cost as the product of the unit price and the number bought is often a complete and accurate model of monetary costs. Some models do not give exact and complete information but approximations that may result from the features of the situation that are reasonably available or of most interest.[MP4] In the business world, the per item price when purchasing a large number of the same item is lower than for the price for a single item. This is important in modeling some situations but may be neglected in others. As another example, consider the linear function describing the cost of purchasing an automobile and gasoline for a number of years

$$C(t) = p + at,$$

where p is the purchase price, t is the number of years, and a is a constant based on assumptions of the cost of gasoline (per gallon), the number of miles driven per year and the fuel efficiency in miles per gallon.[F-BF.1] All of the quantities going into the constant a are estimates and likely will not be constant over time, but a more complex model of gasoline costs and expected driving habits requires information not available and perhaps unnecessary for decision-making. Further, there are costs not included—insurance and maintenance, for example—and the effect of a gasoline-powered automobile on the environment is not considered. However, the simple model may suffice to decide, say, between the purchase of a hybrid version and a gasoline version of an automobile, where the basic differences are in purchase price (hybrids may cost more) and fuel efficiency. Following the advice attributed to Einstein that, "Everything should be made as simple as possible, but not simpler,"[1] we can get good evidence to support the choice of the simple model $C(t) = p + at$, so in this situation this is "as simple as possible," but dropping the purchase price p, or the term at would delete critical information for our decision, based on cost differences. That would delete necessary information, moving us to Einstein's "simpler [than possible]."

Models mimic features of reality. These features are often selected for particular uses. For example, a road map is a model. So

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[1]Probably a paraphrase of "It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience" from "On the Method of Theoretical Physics."

What is a model?

The word "model" can be used as a noun, verb, or adjective. As an adjective, "model" often signifies an ideal, as in "model student." In this progression, "model" will be a noun or a verb.

In elementary mathematics, a model might be a representation such as a math drawing or a situation equation (operations and algebraic thinking), line plot, picture graph, or bar graph (measurement), or building made of blocks (geometry). In Grades 6–7, a model could be a table or plotted line (ratio and proportional reasoning) or box plot, scatter plot, or histogram (statistics and probability). In Grade 8, students begin to use functions to model relationships between quantities.

Models are also used to understand mathematical or statistical concepts. In elementary grades, students use rows of dots or tape diagrams to represent addition and subtraction. Later they use tape diagrams, arrays, and area models to represent multiplication and division. In Grade 6 geometry, nets can represent a three-dimensional mathematical object (e.g., a prism) as well as a design for a real world object (e.g., a gingerbread house). In Grade 8, students use physical models, transparencies, or geometry software to understand congruence and similarity. In Grade 6–8 statistics, simulations help students to understand what can happen during statistical sampling.

In high school, modeling becomes more complex, building on what students have learned in K–8. Representations such as tables or scatter plots are often intermediate steps rather than the models themselves.

[K.OA.2]Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

[MP4] Mathematically proficient students . . . are comfortable making assumptions and approximations . . . realizing that these may need revision later.

[F-BF.1] Write a function that describes a relationship between two quantities.

is a geological map. Features that are important on road maps, e.g., major highways, may not be important on a geological map. The features of the real world situation mimicked by a mathematical model fall into three categories:[2]

- Things whose effects are neglected.

- Things that affect the model but whose behavior the model is not designed to study—inputs or independent variables.

- Things that the model is designed to study—outputs or dependent variables.[3]

These features of a mathematical model are helpful to keep in mind. For example, in the cost function C above, the effect on environment, insurance costs, and maintenance costs are neglected. Inputs are cost of gasoline, miles driven per year, and fuel efficiency rate. The output, or dependent variable, is the cost .

Modeling in K–12

Modeling is critically important, but is not easy. Some idealized, simple modeling problems are needed for learning throughout K–12, but real problems easily available and solvable (perhaps with the assistance of technology). Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena as well as physical phenomena. Situations which are not modeled by simple equations can often be understood by simulation on a calculator, desktop, or laptop, a process which many students will find especially engaging because of its exploratory and open-ended nature. These tools allow for modeling complex real world situations, and most real world situations are complex.

While there is certainly no limit to the sophistication of a model or of the mathematics used in a model, the essence of modeling is often to use humble mathematics in rather sophisticated ways. For example, percentages are often crucial in modeling situations. "Distance equals rate times time" is a powerful idea that is introduced in grade 6 [cite] that nevertheless forms the basis for many useful models throughout high school and beyond. Or as another example, when high school students make an order of magnitude estimate, they may learn a great deal by using only simple multiplication and division. Likewise, statistical modeling in high school might often involve only measures of center and variability, rather than relying on a host of sophisticated statistical techniques. "Back of the envelope" modeling is one of the discipline's most powerful forms.

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[2]Bender, 1978, *An Introduction to Mathematical Modeling*, John Wiley and Sons.

[3]Statistical modeling also involves relationships among variables, but the relationship may be construed as association (e.g., correlation) rather than dependency.

Many situations in the real world involve rate of change, with models that involve a differential equation. Although differential equations are not in the Standards, the interpretation of rates of change[S-ID.7] and the study of functions with base rules of growth[F-LE.1] prepares the way for the study of more sophisticated models in college. Likewise, using probability in modeling greatly extends the scope of real world situations which can be modeled.

News media accounts of topics of current interest often illustrate why modeling and understanding the models of others is important, mostly for informed citizenship. For example, probabilities often are stated in terms of odds in media accounts. Thus, to connect such accounts to school mathematics, students need to know the relationship between the two. Learning to model and understand models is enhanced by seeing the same mathematics or statistics model situations in different contexts.• Media accounts provide those varied contexts in circumstances that require critical thinking. Analyzing these accounts provides opportunities for students to maintain and deepen their understanding of modeling in high school and after graduation.[4]

S-ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

• For example, right triangles are a frequent model for situations that students may initially see as different mathematically, e.g., finding the length of the shadow cast by an upright pole and finding the height of a tree or building. A line fitted to a scatter plot is often used in statistics to model relationships between two measurement quantities. Risk factors are often derived from relative frequency within a single sample.

The Modeling Process

In the Standards, modeling means using mathematics or statistics to describe (i.e., model) a real world situation and deduce additional information about the situation by mathematical or statistical computation and analysis. For example, if the annual rate of inflation is assumed to be 3% and your current salary is $38,000 per year, what is an equivalent salary t years in the future? What salary is equivalent in 10 years? The model is a familiar one to many:

$$S(t) = 38{,}000(1.03^t).$$

This aspect of modeling produces information about the real world situation via the mathematical model, i.e. the real world is understood through the mathematics.

Complex models are often built hierarchically, out of simpler components which can then be artfully joined together to capture the behavior of the complex system. Certain simplifications have become standard based on historical use. For example, the consumer price index (CPI) and the cost of living index (COLI) are commonly cited measures that serve as agreed-upon proxies for important economic circumstances, substituting a single quantity for a more complicated collection of quantities that tend to move as a group. There is even an index of indexes, the index of leading economic indicators. The monthly payments required to amortize a home mortgage over 30 years are computed by summing a geometric series and manipulating the results.[A-SSE.4] Numerous political and economic debates

A-SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

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[4]For further examples, see Dingman & Madison, 2010, "Quantitative Reasoning in the Contemporary World" [two-part article], *Numeracy*, `http://scholarcommons.usf.edu/numeracy/vol3/iss2/`.

Draft, 4 July 2013, comment at **commoncoretools.wordpress.com**.

center on how one measures amounts of money, that is, what units are used. Measuring amounts of money in nominal dollars (dollars-of-the-day) over periods of several years is very different from measuring in constant dollars (the dollar of a particular year). Measuring in percent of gross domestic product (GDP) is also different. Understanding what these are and how to move from one unit to the others is critical in understanding many issues important to personal prosperity and responsible citizenship.

Probability and statistical models abound in news media reports. Complex and heretofore unusual graphics are made possible by technology and in recent years the diversity of graphical models in media accounts has increased enormously. Many of these models and the situations they describe are very important for making decisions about health issues or political circumstances. Political polls model elections themselves,[S-IC.1] and skeptics decry their predictions because they are based on a small sample of all eligible voters. Lack of understanding leads to suspicion and distrust of democratic processes.

S-IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

Modeling in High School

The Modeling Cycle
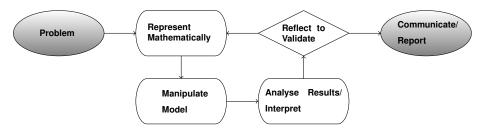
In high school, modeling involves a way of thought different from what students are taught when they learn much of the core K–8 mathematics. It provides experience in approaching problems that are not precisely formulated and for which there is not necessarily a single "correct" answer. Deciding what is left out of a model can be as important as deciding what is put in. Judgment, approximation, and critical thinking enter into the process. Modeling can have differing goals depending on the situation—sometimes the aim is quantitative prediction, for example in weather modeling, and sometimes the aim is to create a simple model that captures some qualitative aspect of the system with a goal of better understanding the system, for example modeling the cyclic nature of predator–prey populations.

Why is modeling difficult? Modeling requires multiple mental activities and significant human skills of abstraction, analysis, and communication. First, a real world situation must be understood in terms familiar to the student. Critical variables must be identified and those that represent essential features are selected. Second, the interpreted situation must be represented—by diagrams, graphs, equations, or tables. Moving from the interpretation to the representation involves reasoning—algebraic, proportional, quantitative, geometric, or statistical. Symbolic manipulation and calculation[A-SSE.3] may follow to produce expressions for the desired quantities. A critical step is now to interpret the quantitative information in terms of the original situation. The quantitative information must be analyzed or synthesized, that is, information is either combined to make

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

some judgment or separated into pieces to do so. During this anal‐ysis or synthesis, assumptions are either made or assumptions are evaluated. At this point, the information obtained is evaluated in terms of the original situation. If the information is unreasonable or inadequate, then the model may need to be modified to re‐start the whole process. If the information is reasonable and adequate, the results are communicated in terms reflecting the original real world context and the information sought by the student. Understanding the limitations of the model involves critical thinking.

This figure is a variation of the figures in the introduction to high school modeling in the Standards.

Diagrams of modeling processes vary. For example, a diagram that focuses on reasoning processes has four components: Descrip‐tion, Manipulation, Translation or Prediction, and Verification.[5] Par‐titioning the modeling process into reasoning components is help‐ful in identifying where reasoning is succeeding or failing. This is important in both assessing student work and guiding instruction. These diagrams of modeling processes are intended as guides for teachers and curriculum developers rather than as illustrations of steps to be memorized by students.

Units and Modeling

Throughout the modeling process, units are critical for several rea‐sons, including guiding the symbolic or numeric calculations.[N-Q.1] Keeping track of units is very helpful in determining if the calcula‐tions are meaningful and lead to the desired results. Units are also critical in the analysis and synthesis and in making or evaluating assumptions, as well as determining reasonableness of answer. For example, if analysis of a cost equation for driving an automobile in‐dicates that a typical driver in the US will drive 5000 miles per year, one should check units to make sure that the gallons are US gallons and the fuel efficiency is in miles per US gallon. Most of the world measures gasoline in liters and distances in kilometers rather than miles. (According to the Federal Highway Administration, the aver‐age number of miles driven per year by US drivers is over 13,000.)

N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

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[5]See Lesh & Doerr, 2003, *Beyond Constructivism: Models and Modeling Perspec‐tives on Mathematics Problem Solving, Learning, and Teaching*, Lawrence Erlbaum Associates, p. 17.

*Draft, 4 July 2013, comment at **commoncoretools.wordpress.com**.*

Units are almost always essential in communicating the results of a model since answers to real world problems are usually quantities, that is, numbers with units. Modeling prior to high school produces measures of attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling such as acceleration, percent of GDP, person–hours, and some measures where the units are not specified and have to be understood in the way the measure is defined.[N-Q.2] For example, the S&P 500 stock index is a measure derived from the quotient of the value of 500 companies now and in 1940–42.

N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.

Modeling and the Standards for Mathematical Practice

One of the eight mathematical practice standards—MP4 Model with mathematics—focuses on modeling and modeling draws on and develops all eight. This helps explain why modeling with mathematics and statistics is challenging. It is a capstone experience, the proof of the pudding. To embody this, students might complete a capstone experience in modeling.

Make sense of problems and persevere in solving them (MP1) begins with the essence of problem solving by modeling: "Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution." Solving a real life problem in a non-mathematical context by mathematizing (i.e. modeling) requires knowing the meaning of the problem and finding a mathematical representation. Later in this standard, "Younger students might rely on using concrete objects or pictures [i.e. models] to help conceptualize and solve a problem."

Reason abstractly and quantitatively (MP2) includes two critical modeling activities. The first is "the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate," and the second is that "Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved." Decontextualizing and representing are fundamental to problem solving by modeling.

Construct viable arguments and critique the reasoning of others (MP3) notes that mathematically proficient students "reason inductively about data, making plausible arguments that take into account the context from which the data arose"—the data being the model educed from some context. Further, "Elementary students can construct arguments using concrete referents (i.e. models) such as objects, drawings, diagrams, and actions." Discussing the validity of the model and the level of uncertainty in the results makes use of these skills.

Use appropriate tools strategically (MP5) notes that "When making mathematical models, [mathematically proficient students] know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data." Simulation provides an important path to explore the conse-

quences of a model, and to see what happens when parameters of the model are varied.

Attend to precision (MP6). Here the most important consideration of modeling is to "express numerical answers with a degree of precision appropriate for the problem context" and in appropriate units. For example, if one is modeling the annual debt or surplus (there were no surpluses) in the US federal budget over the decade 2001–2010, then common options for a unit are nominal dollars, constant dollars, or percent of GDP. The degree of precision appropriate for understanding the model is to the nearest billion dollars (or nearest tenth percent of GDP) or perhaps the nearest ten billion dollars (or nearest percent of GDP).$^{\text{N-Q.3}}$ Beyond accuracy, modeling raises the issue of uncertainty—how likely are the quantities we want to model to be within a certain range. How much do features the model neglects affect accuracy and uncertainty?

Look for and make use of structure (MP7). Here, looking closely at a real world situation to discern relationships between quantities is critical for mathematical modeling. Students look for patterns or structure in the situation, for example, seeing the side of a right triangle when a shadow is cast by an upright flagpole as part of a right triangle or seeing the rise and run of a ramp on a staircase.

Look for and express regularity in repeated reasoning (MP8). Modeling activities often involve multistep calculations and the whole modeling cycle may need to be repeated. Here, mathematically proficient students "continually evaluate the reasonableness of their intermediate results" and "maintain oversight of the process" (in this case, the modeling process).

N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Modeling and Reasonableness of Answers

Continually evaluating reasonableness of intermediate results in problem solving is important in several of the standards for mathematical practice. Doing this often requires having reference values, sometimes called anchors or quantitative benchmarks, for comparison. Joel Best, in his book Stat-Spotting,[6] lists a few quantitative benchmarks necessary for understanding US social statistics: the US population, the annual birth and death rates, and the approximate fractions of the minority subpopulations. Without these reference values, an answer of 27 million 18-year-olds in the US population may seem reasonable. Such benchmarks for other measures are helpful, providing quick ways to mentally check intermediate answers while solving multistep problems. For example, it is very helpful to know that a kilogram is approximately 2 pounds, a meter is a bit longer than a yard, and there are about 3 liters in a gallon. This kind of quantitative awareness can be developed with prac-

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[6] 2008, *Stat-Spotting: A Field Guide to Identifying Dubious Data*, University of California Press.

tice, and easily expanded with the immense amount of information readily available from the internet.

Statistics and Probability

Specific modeling standards appear throughout the high school standards indicated by a star symbol (★). About one in four of the standards in Number and Quantity, Algebra, Functions, and Geometry have a star, but the entire conceptual category of Statistics and Probability has a star. In statistics, students use statistical and probability models—whose data and variables are often embodied in graphs, tables, and diagrams—to understand reality. Statistical problem solving is an investigative process designed to understand variability and uncertainty in real life situations. Students formulate a question (anticipating variability), collect data (acknowledging variability), analyze data (accounting for variability), and interpret results (allowing for variability).[7] The final step is a report.

Much of the study of statistics and probability in Grades 6–8 concerns describing variability, building on experiences with categorical and measurement data in early grades (see the progressions for these domains). In high school the focus shifts to drawing inferences—that is, conclusions—from data in the face of statistical uncertainty. In this process, analyzing data may have two steps: representing data and fitting a function (often called the model) which is intended to capture a relationship of the variables. For example, bivariate quantitative data might be represented by a scatter plot and then the scatter plot is modeled as a linear, quadratic, or logarithmic function. A probability distribution might be represented as a bar graph and then the bar graph is modeled by an exponential function. See the high school Statistics and Probability Progression for examples.

Because the Statistics and Probability Progression for high school is also a modeling progression, the discussion here will only note statistics and probability standards when they are related to modeling standards in one of the other conceptual categories.

Developing High School Modeling

In early grades, students use models to represent addition and subtraction relationships among quantities such as 2 apples and 3 apples, and to understand numbers and arithmetic. Concrete models, drawings, numerical equations, and diagrams help to explain arithmetic as well as represent addition, subtraction, multiplication, and division situations described in the Operations and Algebraic Thinking Progression. Later, students use graphs and symbolic equations

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[7]See the American Statistical Association's 2007 *Guidelines for Assessment and Instruction in Statistics Education*, Alexandria, VA: American Statistical Association, 2007, pp. 11–15, `http://www.amstat.org/education/gaise`.

to represent relationships among quantities such as the price of n apples where p is the price per apple. In Grade 8, calculating and interpreting the concept of slope may, in various contexts, draw on interpreting subtraction as measuring change or as comparison, and division as equal partition or as comparison (see Tables 2 and 3 of the Operations and Algebraic Thinking Progression). Creation of exponential models builds on initial understanding of positive integer exponents as a representation of repeated multiplication, while identifying the base of the exponential expression from a table requires the unknown factor interpretation of division. Extension of an exponential model from a geometric sequence to a function defined on the real numbers builds on the understanding of rational and irrational numbers developed in Grades 6–8 (see The Number System Progression).

By the beginning of high school, variables and algebraic expressions are available for representing quantities in a context. Modeling in high school can proceed in two ways. First, problems can focus directly on the concepts being studied, i.e., situations such as the path of a projectile which are modeled by quadratic equations can be a part of the study of quadratic equations. This is the traditional path followed by having a section of word problems at the end of a lesson. A second, more realistic, way to develop modeling is to utilize situations that can become more complex as more mathematics and statistics are learned.[8] It is unlikely that one situation can be used throughout high school modeling, but some situations can be increased in complexity (examples are given in this progression). Modeling with mathematics in high school begins with linear and exponential models and proceeds to representing more complex situations with quadratics and other polynomials, geometric and trigonometric models, logic models such as flow charts, diagrams with graphs and networks, composite functional models such as logistic ones, and combinations and systems of these. Modeling with statistics and probability (that is, as noted earlier, essentially all of statistics and probability) is detailed in the progression for that conceptual category.

Linear and Exponential Models

In high school, the most commonly occurring relationships are those modeled by linear and exponential functions. Examples abound. The number of miles traveled in t hours by an automobile at a speed of 30 miles per hour is $30t$ and the amount of money in an account earning 4% interest compounded annually after 3 years is $P(1.04)^3$ where P is the initial deposit. Students learn to identify the referents of

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[8]For examples, see Schoen & Hirsch, "The Core-Plus Mathematics Project: Perspectives and Student Achievement," and Senk, "Effects of the UCSMP Secondary School Curriculum on Students' Achievement" in Senk & Thompson (Eds.), 2003, *Standards-Based School Mathematics Curricula: What Are They? What Do Students, Learn?*, Lawrence Erlbaum Associates.

symbols within expressions (MP2), e.g., 30 is the speed (or, later, velocity), t is the time in hours, and to abstract distance traveled as the product of velocity and time.

In Grade 8, students learned that functions are relationships where one quantity (output or dependent variable) is determined by another (input or independent variable).[8.F.1] In high school, they deepen their understanding of functions, learning that the set of inputs is the domain of the function and the set of outputs is the range.[F–IF.1] For example, the car traveling 30 miles per hour travels a distance d in t hours is expressed as a function

$$d(t) = 30t.$$

Students learn that when a function arises in a real world context a reasonable domain for the function is often determined by that context.

Students learn that functions provide ways of comparing quantities and making decisions. For example, a more fuel–efficient automobile costs $3000 more than a less fuel efficient one, and $500 per year will be saved on gasoline with the more efficient car. (This can be made precisely realistic by using data, say, from comparing a hybrid version to a gasoline version of an automotive model.) A graph of the net savings function

$$S(t) = 500t - 3000$$

(see margin) will have a vertical intercept at $S = -3000$ and a horizontal intercept at $t = 6$. Students learn that the horizontal intercept, or the zero of the function, is the break–even point, that is, by year 6 the $3000 extra cost has been recovered in savings on gasoline costs.[F-LE.5]

As students learn more about comparing functions that have domains other than the nonnegative integers, this example can be increased in complexity.[10] The buyer has the option of paying the extra $3000 and saving money on gasoline or placing the $3000 in a savings account earning 4% per year compounded yearly. One option yields the net savings

$$S(t) = 500t - 3000$$

while the other yields amount

$$A(t) = 3000(1.04^t).$$

Students compare $S(t)$ and $A(t)$ by graphs or tables over some number of years, the domain of the functions.[F-IF.9] The expected time the buyer will drive the car determines a reasonable domain. A table of values for A and S (shown in the margin) over years 1 to 20 is likely to be sufficient for comparing the functions,[F-IF.6] or, later when

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[10]Example from Madison, Boersma, Diefenderfer, & Dingman, 2009, *Case Studies for Quantitative Reasoning*, Pearson Custom Publishing.

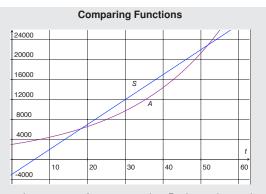*Draft, 4 July 2013, comment at* **commoncoretools.wordpress.com**.

[8.F.1]Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[9]

[F-IF.1] Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

[F-LE.5] Interpret the parameters in a linear or exponential function in terms of a context.

Comparing Functions

Year t	$S(t)$	$A(t)$	Year t	$S(t)$	$A(t)$
0	-3000	3000	10	2000	4441
1	-2500	3120	11	2500	4618
2	-2000	3245	12	3000	4803
3	-1500	3375	13	3500	4995
4	-1000	3510	14	4000	5195
5	-500	3650	15	4500	5403
6	0	3796	16	5000	5619
7	500	3948	17	5500	5844
8	1000	4106	18	6000	6077
9	1500	4270	19	6500	6321

Outcomes for two scenarios at year t. If the hybrid is purchased, its savings on gasoline costs plus the difference in price between hybrid and gasoline models is shown as $S(t)$. If the gasoline model is purchased and the price difference is invested, the amount of the investment is $A(t)$.

Comparing Functions

Comparing outcomes for two scenarios: Buying and operating a hybrid automobile vs buying and operating a gasoline automobile and investing the difference in their prices.

[F-IF.9] Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

[F-IF.6] Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

non-integer domains are understood, the graphs of S and A over the interval $[0,20]$ will give considerable information (see the margin). The vertical intercepts of the two graphs and their two points of intersection are interpreted in the context of the problem. Analysis of the key features of the two graphs[F-IF.4] provides opportunities for students to compare the behaviors of linear and exponential functions. Students observe the average rates of change of the two functions over various intervals[F-IF.6] and see why the exponential function values will eventually overtake the linear function values and remain greater beyond some point. Students can now report on the information that will influence an economic decision by relating the behavior of the graphs to the comparative savings.

Students can again question the assumptions underlying the models of the two savings functions. What is the effect if the cost of gasoline changes? What is the effect if the number of miles driven changes? What will be the results of periodically (say, annually) placing the savings on gasoline costs in the savings account earning 4% per year compounded yearly? This latter option changes the linear model to a second exponential model, starts with a sum of a geometric series, which can be expressed either recursively or with an explicit formula,[F-BF.2] and points to the advantages of rewriting the sum of exponential expressions as a single exponential expression.[A-SSE.3c] This reinforces that algebraic re-writing of expressions is helpful, sometimes essential, to achieve comprehensible and usable models.

In the above example, students learn to question why the two scenarios have a $6000 difference at year 0. Students might argue that the $3000 is being invested two ways—one way is investing in the automobile and one way is placing in a savings account. The question then becomes: Which investment produces the most returns? That would make both functions be 0 at time 0. Is it more reasonable to note that the difference is $3000 and not $6000? In that case the graphs look like the ones here, and the table above is altered by reducing each entry for $A(t)$ by 3000.

Students learn that some initial representations and calculations can be done by hand,[F-IF.7] say, the graph of $S(t) = 500t - 3000$ and its key features. With iterations of the modeling cycle, the model becomes more complicated. Specific outputs of the functions can be calculated by hand, but technology is essential to understand the overall situation.

Students learn to distinguish between scenarios like the one above where two (or more) equations or functions give different results based on different assumptions about the situation and scenarios where the two (or more) equations (possibly, inequalities) or functions express relationships among the quantities of interest under the same assumptions. The latter scenarios are modeled by a system of equations or inequalities. A system of equations imposes multiple conditions on a situation, one for each of the equations. Solutions to systems must satisfy each of the equations. For example,
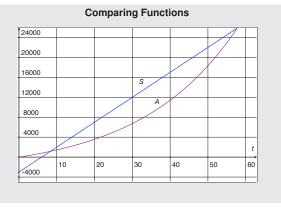
F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

A-SSE.3c Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

c Use the properties of exponents to transform expressions for exponential functions.

Comparing Functions

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a system of two linear equations[A-CED.3] will model the speed that you can row a boat with no current and the speed of the current provided you know the speed of the boat as you row with the current and the speed you can row against the current. Students learn how to describe situations by systems of two or three equations or inequalities and to solve the systems using graphs, substitution, or matrices. Students learn to detect if a system of equations is consistent, inconsistent, or independent.

Later, as students are challenged to develop more complex models, the processes of solving systems of equations are used to synthesize and develop new relationships from systems of equations that model a situation. Thus, students are challenged to use substitution to combine parametric equations and giving the spatial coordinates of a projectile as a function of time into a single relation modeling the path of the projectile, or to incorporate a constraint on the volume into a formula giving the cost of a cylindrical can as a function of the radius.

Counting, Probability, Odds and Modeling

In Grades 7 and 8, students learned about probability and analysis of bivariate data. In high school, students learn the meanings of correlation and causation. Correlation, along with standard deviation, is interpreted in terms of a linear model of a data set. Students distinguish in models of real data the difference between correlation and causation.[S-ID.7 ,S-ID.8 ,S-ID.9]

Students' intuitions, affected by media reports and the surrounding culture (cf. Nobel Laureate Daniel Kahnemann's Thinking Fast and Slow), sometimes conflict with their study of probability. Unusual events do occur and unconditional theoretical probabilities are based on what will happen over the long term and are not affected by the past—the probability of a head on a coin flip is $\frac{1}{2}$ even though each of the seven previous flips resulted in a head. Students learn how to reconcile accounts of probability from public and social media with their study of probability in school. For example, they learn the intriguing difference between conspiracy and coincidence.

Relating the study of probability to everyday language and feelings is important. Students learn about interpreting probabilities as "how surprised should we be?" Students learn to understand meanings of ordinary probabilistic words such as "unusual" by examples such as: "The really unusual day would be one where nothing unusual happens" and "280 times a day, a one-in-a-million shot is going to occur," given that there were approximately 280 million people in the US at the time. Coincidence is described as "unexpected connections that are both riveting and rattling."●

Because probabilities are often stated in news media in terms of odds against an event occurring, students learn to move from probabilities to odds and back. For example, if the odds against a horse winning a race are 4 to 1, the probability that the horse

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

S-ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S-ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

S-ID.9 Distinguish between correlation and causation.

● "How surprised should we be?" is attributed to statistician Bradley Efron. "The really unusual day" and other examples are attributed to mathematician Persi Diaconis. See Belkin, 2002, "The Odds of That," New York Times Magazine, `http://www.nytimes.com/2002/08/11/magazine/11COINCIDENCE.html`.

will win is estimated to be $\frac{1}{1+4}$. If the probability that another horse will win is 0.4 then the odds against that horse winning is the probability of not winning, 0.6, to the probability of winning, 0.4, written as 0.6–0.4 or, equivalently, 3–2 or 3:2 and read as "3 to 2." The equivalence of 0.6–0.4 and 3–2 highlights the fact that odds are ratios of numbers, where the numerator and denominators convey meaning. Students learn that the sum of the probabilities of mutually exclusive events occurring cannot exceed 1, but that they sometimes do in media reports where odds and probabilities are approximated for simplicity.

Counting to determine probabilities continues into high school, and student learning is reinforced with models. For example, the birthday problem provides rich learning experiences and shows students some outcomes that are not intuitively obvious. Counting the number of possibilities for n birthdays yields an exponential expression 366^n, and counting of the number of possibilities for n birthdays all to be different yields a permutation P_n^{366}. The quotient is the probability that n randomly chosen people will all have different birthdays, yielding the probability of at least one birthday match among n people. The often surprising result that when $n = 23$ there is approximately a 50-50 chance (probability of 0.5 or 50–50 odds) of having a match. Students learn that the function

$$P(n) = \frac{P_n^{366}}{366^n}$$

models the probability of having no birthday match for n randomly chosen people, and $1 - P(n)$ is the probability of at least one birthday match. The results can be modeled by a spreadsheet revealing the probabilities for $n = 2$ to $n = 367$. Students learn that it requires at least 367 people to have a probability of 1 of a birthday match and also learn about the behavior of technology in that the spreadsheet values for the probability of at least one match become 1 (or at least report as 1) for values of n less than 367. Students calculating $P(n)$ using hand held calculators learn that for values of n of approximately 40, many hand-held calculators cannot compute the numerators and denominators because of their size. This provides an opportunity to learn that rewriting the quotient of the two, too large numbers as the product of a sequence of simpler quotients allows the calculator to compute the sequence of quotients and then take their product. On TI calculators this takes the form

$$\mathrm{Prod}\left(\mathrm{Seq}\left(\frac{x}{366}, x, 366, 366 - n + 1, -1\right)\right),$$

that is, the product of the sequence

$$\left\{\frac{366}{366}, \frac{365}{366}, \dots, \frac{366 - n + 1}{366}\right\},$$

which students learn is a product of probabilities. Students learn that more complex questions can be asked about birthday matches.

For example, what is the probability of having exactly one, or exactly two matches of birthdays among n people?

Key Features to Model

Students learn key features of the graphs of polynomials, rational functions, exponential and logarithmic functions, and modifications such as logistic functions to help in choosing a function that models a real life situation. For example, logistic functions are used in modeling how many students get a certain problem on a test right, and thereby are used in evaluating the difficulty of a problem on a standardized test. A quadratic function might be considered as model of profit from a business if the profit has one maximum (or minimum) over the domain of interest. An exponential function may model a population over some portion of the domain, but circumstances may constrain the growth over other portions. Piecewise functions are considered in situations where the behavior is different over different portions of the domain of interest.
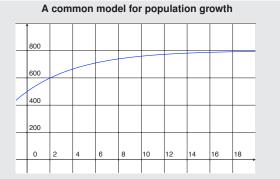
Students learn that real life circumstances such as changes in populations are constrained by various conditions such as available food supply and diseases. They learn that these conditions prevent populations from growing exponentially over long periods of time. A common model for growth of a population P results from a rate of change of P being proportional to the difference between a limiting constant and P, as in Newton's law of cooling. This constrained exponential growth results in P being given by the difference between the limiting constant and an exponentially decaying function. For example, the margin shows the graph of a population that is initially 500 and approaches a limiting value of 800. Another common population growth model results from logistic functions where the rate of growth of P is proportional to the product of $P(a - P)$ for some constant a.

Students learn to look at key features of the graphs of models of constrained exponential growth (or decay) and logistic functions (intercepts, limiting values, and inflection points) and interpret these key features into the circumstances being modeled.[F–IF.4]

Formulas as Models

Formulas are mathematical models of relationships among quantities. Some are statements of laws of nature—e.g., Ohm's Law, $V = IR$, or Newton's law of cooling—and some are measurements of one quantity in terms of others—e.g., $V = \pi r^2 h$, the volume of a right circular cylinder in terms of its radius and height.[G-MG.2] Students learn how to manipulate formulas to isolate a quantity of interest. For example, if the question is to what depth will 50 cubic feet of a garden mulch cover a bed of area 20 square feet, then the formula $h = \frac{V}{A}$ where h is the depth, V is the volume, and A is the area, is an appropriate form.[A-CED.4] If one wants a depth of 6

*Draft, 4 July 2013, comment at **commoncoretools.wordpress.com**.*

A common model for population growth

The function shown in the graph is $P(t) = 800 - 300e^{-kt}$ where k is a positive constant, the solution to
$$\frac{dP(t)}{dt} = k(800 - P(t)).$$

F–IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

G–MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

A–CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

inches, then the form would be the appropriate for finding how much mulch to buy. Students learn that the shape of the bed (modeled as the base of a cylinder) does not matter, an application of Cavalieri's principle;[G-GMD.1] volume is the product of the area and the height.[G-GMD.3]

Formulas that are models may sometimes be readily transformed into functions that are models. For example, the formula for the volume of a cylinder can be viewed as giving volume as a function of area of the base and the height, or, rearranging, giving the area of the base as a function of the volume and height. Similarly, Ohm's law can be viewed as giving voltage as a function of current and resistance. Newton's law of cooling states that the rate of change of the temperature of a cooling body is directionally proportional to the difference between the temperature of the body and the temperature of the environment, i.e., the ambient temperature.[F-BF.1b] This is another example of constrained exponential growth (or decay). The solution of this change equation (a differential equation) gives the temperature of the cooling body as a function of time.

In Grade 7 students learned about proportional relationships and constants of proportionality.[7.RP.2] These surface often in high school modeling. Students learn that many modeling situations begin with a statement like Ohm's law or Newton's law of cooling, that is, that a quantity of interest, I, is directly proportional to a quantity, V, and inversely proportional to a quantity, R, i. e. I is given by the product of a constant and $\frac{V}{T}$. Newton's law of cooling is stated as a proportionality giving the rate of change of the temperature at a given moment as a product of a constant and the difference in the temperatures—this can be used in forensic science to estimate the time of death of a murder victim based on the temperature of the body when it is found.

Right Triangle and Trigonometric Models Students learn that many real world situations can be modeled by right triangles. These include areas of regions that are made up of polygons, indirect measurement problems, and approximations of areas of non-polygonal regions such as circles. Examples are areas of regular polygons, height of a flag pole, and approximation of the area of a circle by regular polygons. Prior to extending the domains of the trigonometric functions by defining them in terms of arc length on the unit circle, students understand the trigonometric functions as ratios of sides of right triangles. These functions, paired with the Pythagorean Theorem, provide powerful tools for modeling many situations.[G-SRT.8]

When the domains of the trigonometric functions are extended beyond acute angles,[F-TF.2] the reasons that these functions are called "circular functions" become clearer. Many situations involving circular motion can be modeled by trigonometric functions. The example below uses trigonometric functions and vector-valued functions. For example, prior to GPSs, this is how sailors determined their latitude.

G-GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

G-GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

F-BF.1b Write a function that describes a relationship between two quantities.
  b Combine standard function types using arithmetic operations.

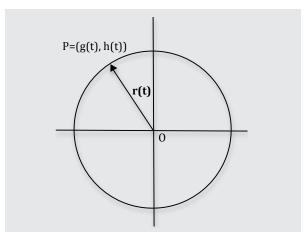**7.RP.2** Recognize and represent proportional relationships between quantities.

G-SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

F-TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Where the Modeling Progression might lead

As mentioned earlier, modeling in high school becomes more complex and powerful as more mathematics and statistics are used to describe real life circumstances. As students learn more, they learn to use new concepts to extend simpler models previously studied. Although a high school modeling problem is not likely to incorporate all of high school mathematics, there are models that incorporate many concepts and extend beyond the high school mathematics described in the Standards. The motion of communication satellites around the earth or the motion of an object spinning rapidly in a circle by holding one end of a string with the other attached to the object can be modeled as a point traversing a circle. The object (at point P) is accelerated toward the center (O) of the circular path and the magnitude of the acceleration is constant.

The position vector $\vec{r}(t)$ joining O and P at time t is given by $\vec{r}(t) = g(t)\vec{i} + h(t)\vec{j}$ where $\vec{i} = (1,0)$ and $\vec{j} = (0,1)$ are unit vectors. By considering the geometry and the physics of the situation, one can show that there are functions $g(t)$ and $h(t)$ (twice differentiable, giving the velocity and acceleration vectors of the motion) satisfying the conditions of the model. Noting the similarities of the conditions on g and h to the behavior of the trigonometric functions $\sin(t)$ and $\cos(t)$ one can show that indeed the vector function describes uniform circular motion for an object P on a circle of radius R and a constant magnitude of acceleration.[F-TF.5]

Adapted from Usiskin, Peressini, Marchisotto, & Stanley, 2003, Mathematics for High School Teachers: An Advanced Perspective, *Pearson Prentice Hall, pp. 469–474.*

[F-TF.5] Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Notes

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